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EFFECT OF LONGITUDINAL HEAT CONDUCTION
ON ROTARY REGENERATORS

GUSTAVO D. BAHNKE

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EFFECT OF LONGITUDINAL HEAT CONDUCTION
ON ROTARY REGENERATORS

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Gustavo D. Bahnke

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EFFECT OF LONGITUDINAL HEAT CONDUCTION
ON ROTARY REGENERATORS

by

Gustavo D. Bahnke
//
Lieutenant, Chilean Navy

Submitted in partial fulfillment of
the requirements for the degree of
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1962

EFFECT OF LONGITUDINAL HEAT CONDUCTION
ON ROTARY REGENERATORS

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This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

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MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School

ABSTRACT

The general differential equations describing the behavior of the rotary regenerator including longitudinal heat conduction in the direction of fluid flow are sufficiently complicated to preclude a complete analytical solution. A few solutions found in the literature restricted to very special cases are discussed.

A numerical finite difference method is presented which will determine the effect of longitudinal heat conduction in rotary regenerators for steady state conditions. In the development no assumptions are made which would restrict the range of parameters for which the analysis would be applicable.

The conduction effect on the regenerator effectiveness was evaluated with the view of obtaining results most useful for the gas turbine regenerator problem, however, these results may also be used for other regenerator problems.

A CDC 1604 digital computer was used to carry out the computations. The results are presented graphically and in tabular form, employing a suitable set of non dimensional parameters. The range of parameters which have been covered are:

$$0.5 \leq C_{\min}/C_{\max} \leq 1.0$$

$$1.0 \leq C_r/C_{\min} \leq 10.0$$

$$1.0 \leq NTU_o \leq 20.0$$

$$0.25 \leq (hA)^* \leq 1.0$$

$$0 \leq \lambda \leq 0.2$$

$$0.15 \leq As^* \leq 1.3$$

The author expresses his appreciation to C. P. Howard, Associate Professor, for his direction and encouragement in this work.

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NOMENCLATURE

- A = heat transfer area on side designated by subscript, sq ft
- A_{s_k} = solid area available for longitudinal heat conduction on the side of C_{max} , sq ft
- A_{s_n} = solid area available for longitudinal heat conduction on the side of C_{min} , sq ft
- A_s = total solid area available for longitudinal heat conduction, sq ft
- C = heat capacity rate (Wc) of fluid or rotor matrix according to subscript, Btu/hr, deg F
- c = specific heat of fluid (constant pressure) or rotor matrix material depending on subscript, Btu/lb, deg F
- h = unit conductance for thermal convection heat transfer, Btu/hr, deg F, sq ft
- k = unit thermal conductivity, Btu/hr, sq ft, deg F/ft
- L = total length of the matrix in the direction of fluid flow, ft
- N = number of subdivisions according to subscript
- Q = heat transfer rate, Btu/hr
- T = temperature of matrix, deg F
- $T_{x(i,j)}$ = subscripted matrix temperature on the side of C_{max} , deg F
- $T_{n(f,g)}$ = subscripted matrix temperature on the side of C_{min} , deg F
- t = temperature of fluid, deg F
- $t_{x(i,j)}$ = subscripted fluid temperature on the side of C_{max} , deg F
- $t_{n(f,g)}$ = subscripted fluid temperature on the side of C_{min} , deg F
- W = mass flow rate of fluid (lb/hr) or matrix (lb, rev/hr), according to subscript
- y = distance of a point from the cold end, measured in units of total length of matrix, $0 \leq y \leq L$

τ = time measured in units of one half period

A_1, A_2, \dots, A_4

C_1, C_2, \dots, C_4

D_1, D_2, \dots, D_4 = constants.

E_1, E_2, \dots, E_4

F_1, F_2, \dots, F_4

Subscripts.-

avg = average

c = cold side

h = hot side

i = inlet

min - minimum magnitude

max - maximum magnitude

n - side of C_{\min}

o = outlet

r = rotor

x = side of C_{\max}

∞ = subscript on e to indicate value extrapolated to an infinite number of elements.

Dimensionless parameters.-

e = exchanger heat transfer effectiveness, ratio of actual to thermodynamically limited maximum possible heat transfer rate

$$\frac{\Delta e}{e} = \frac{e_{\lambda=0} - e_{\lambda}}{e_{\lambda=0}} = \text{longitudinal heat conduction effect}$$

C_{\min}/C_{\max} = Capacity rate ratio of fluid flow streams

C_r/C_{\min} = capacity rate ratio of rotor matrix to minimum fluid capacity rate

$(hA)^* = (hA)_n/(hA)_x$, conductance ratio

$NTU = (hA)/C$, number of transfer units on side designated by subscript

$NTU_o = NTU_n \left[\frac{1}{1 - C_r} \right]$, over-all number of transfer units

$\lambda = \frac{h A_n}{C_{\max}}$, conduction parameter on side of C_{\max}

$\lambda_n = \frac{h A_n}{C_{\min}}$, conduction parameters on side of C_{\min}

$\lambda = \frac{h A_n}{L}$, total conduction parameter

$As^* = As_n/As_x$, conduction area ratio, ratio of solid areas available for heat conduction in the direction of fluid flow

I. Introduction

The regenerative type of heat exchanger has been subjected to extensive treatment within the last thirty years or so and contributions to the theory of regenerators have been made by many writers. The most questionable of the idealizations made in the development of the theory is that of zero matrix conductivity in the direction of fluid flow. The loss due to heat conduction in the matrix material is specially important in the case of high performance regenerators in which a small reduction in the effectiveness will result in a large increase of the heat transfer areas to compensate for the loss. Another important factor is the modern trend toward smaller sizes, since the loss due to conduction of heat in the matrix obviously increases with decreasing length.

Schultz [2]¹ solved the problem including conduction for the special case of the balanced regenerator and very large values of C_r/C_{\min} (i.e., the capacity rate ratio of rotor matrix to minimum fluid capacity rate). In this case the system of partial differential equations defining the problem for the steady state are reduced to ordinary differential equations and the problem can be solved analytically.

Hahnemann [3] extended the solution to the unbalanced regenerator (i.e., unequal partitions of the cross sections and unequal heat transfer coefficients together with unequal capacity rate ratios of the fluid streams) but still for an infinitely large value of C_r/C_{\min} .

London has shown, as reported by Lambertson [1], that a correction factor for conductivity in the direction of fluid flow for approximately

¹Number in brackets refers to bibliography on page 21

equal fluid capacities, of the form:

$$\frac{\Delta e}{e} = \frac{R A}{C L}$$

should result in a somewhat pessimistic prediction of the reduction in effectiveness.

It is the objective of this thesis to develop and present a finite difference numerical analysis with solutions to solve for the conduction effect on effectiveness for the rotary regenerator. Since there are six dimensionless parameters required to specify behavior in the most general case, the ranges of the parameters covered in the solutions will be those of most interest to the gas turbine regenerator, however, the analysis will place no restrictions on these parameters.

2. Method

The conventional idealizations and boundary conditions assumed in the derivation of the governing differential equations, including the heat conduction in the direction of fluid flow [4], are the following:

(See Fig. 1a)

- a. The thermal conductivity of the matrix material is zero in the direction of matrix metal flow. It has a finite value in the direction of fluid flow, and is infinite in the other direction normal to the fluid flow.
- b. The specific heats of the two fluids and matrix material are constant with temperature.
- c. No leakage of the fluids occurs either due to direct leakage or carry over, and each fluid flow is unmixed.
- d. The convective conductance between the fluid and the matrix are constant with flow length.
- e. The fluids pass in counterflow directions.
- f. Entering fluid temperatures are uniform over the flow inlet cross section and constant with time.
- g. The matrix temperature gradient in the direction of fluid flow is zero at the ends.
- h. Regular periodic conditions are established for all matrix elements, i.e., steady state condition.

The differential equations derived by Schultz [2] are given in Appendix 1.

It was suggested by Desbarrere and carried out by Lambertson [1] for the case of no conduction, that a rotary regenerator could be divided

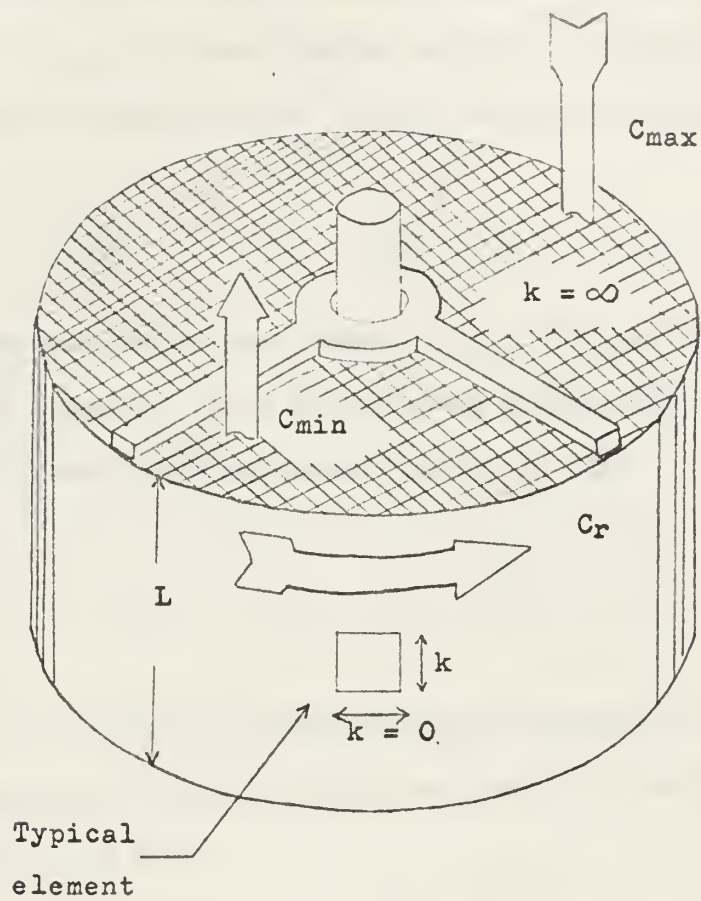


FIG. 1a ILLUSTRATIVE MATRIX ARRANGEMENT
AND FLUID FLOW

into finite elements as represented schematically in Fig. 1b. This same method can be extended to the case with conduction. If the elements are considered to be fixed in space, each element can be regarded as a cross flow heat exchanger with a gas stream and a metal stream. For an element on the side of C_{\max} , the heat transfer rate by convection is equal to the rate of change in enthalpy of the fluid across the element,

$$\frac{(hA)_x}{N_s N_r} \Delta T_{\text{avg}} = \frac{C_{\max}}{N_s} [t_{x(x,y)} - t_{x(x+y,y)}] \quad (1)$$

where ΔT_{avg} represents the mean temperature difference between the fluid and the matrix element. For a small enough element the arithmetic mean temperature difference may be assumed valid, so that

$$\Delta T_{\text{avg}} = \frac{1}{2} [t_{x(x,y)} + t_{x(x+y,y)}] - \frac{1}{2} [T_{x(x,y)} + T_{x(x+y,y)}]$$

where the fluid and matrix temperatures are subscripted to indicate the average temperatures across the inlet and outlet of the element for simple unmixed cross flow.

Considering an energy balance for the element, the energy transferred to the element by convection plus energy transferred to the element by conduction less the energy transferred out of the element by conduction must equal the energy stored in the element:

$$\begin{aligned} \frac{C_{\max}}{N_s} [t_{x(x,y)} - t_{x(x+y,y)}] + k \frac{A_s}{N_c} \frac{[T_{x(x,y)} + T_{x(x+y,y)}] - [T_{x(x,y)} + T_{x(x+y,y)}]}{\frac{2L}{N_r}} \\ - k \frac{A_s}{N_c} \frac{[T_{x(x,y)} + T_{x(x+y,y)}] - [T_{x(x,y)} + T_{x(x+y,y)}]}{\frac{L}{N_r}} = \frac{C_r}{N_r} [T_{x(x,y)} - T_{x(x+y,y)}] \end{aligned} \quad (2)$$

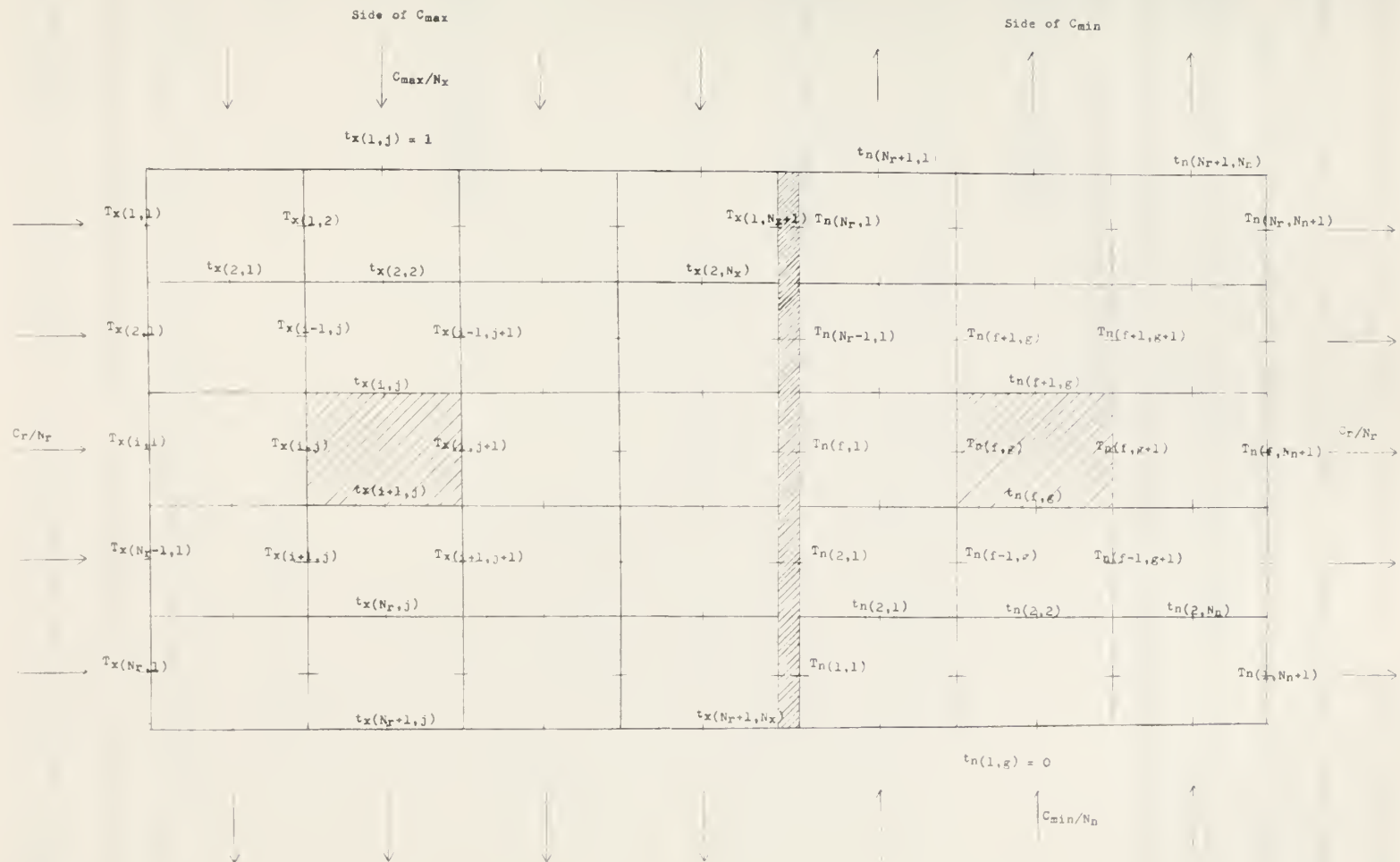


FIG. 1b SCHEMATIC REPRESENTATION OF A ROTARY REGENERATOR

By using equations (1) and (2) the outlet temperatures for an element on the side of C_{\max} may be solved for in terms of the remaining temperatures and the result expressed in dimensionless form (Appendix 1).

Since the longitudinal heat conduction is zero at the ends, the energy balance equation is different from equation (2) for the elements of the first and last row. Three different expressions for the matrix outlet temperatures are obtained:

First row elements

$$T_{x(1,j)} = P T_{x(1,j)} - D T_{x(1,j)} + D_s [T_{x(1,j)} + T_{x(1,j+1)}] \quad (3)$$

Middle row elements

$$T_{x(i,j)} = P T_{x(i,j)} - D T_{x(i,j)} + D_s [T_{x(i,j)} + T_{x(i,j+1)}] \\ = T_{x(i,j)} + [T_{x(i,j+1)}] \quad (4)$$

Last row elements

$$T_{x(n,j)} = P T_{x(n,j)} - D T_{x(n,j)} + D_s [T_{x(n,j)} + T_{x(n,j-1)}] \quad (5)$$

By solving for the fluid outlet temperature on the side of C_{\max} in terms of the matrix outlet temperature from equation (1), only one expression is obtained, which holds for all the elements on the side of C_{\max} ,

$$T_{x(i+1),y} = [D_2 t_{x(i,y)} + D_1] T_{n,y} = T_{x(i+1),y} \quad (6)$$

Similarly, for an element on the side of C_{\min}

$$\frac{C_{\min}}{N_n N_p} \Delta T_{avg} = \frac{C_{\min}}{N_n} [T_{n(i+1),y} - T_{n(i,y)}] \quad (7)$$

with

$$\Delta T_{avg} = \frac{1}{2} [T_{n(i,y)} + T_{n(i+1),y}] - \frac{1}{2} [T_{x(i,y)} + T_{x(i+1),y}]$$

Energy balance:

$$\begin{aligned} \frac{C_{\min}}{N_n} [T_{n(i+1),y} - T_{n(i,y)}] + \lambda \frac{A}{N_n} \frac{[T_{n(i,y)} - T_{x(i,y)}] - [T_{n(i+1),y} - T_{x(i+1),y}]}{\frac{L}{N_n}} \\ - \lambda \frac{A}{N_p} \frac{[T_{n(i,y)} + T_{n(i+1),y}] - [T_{x(i,y)} + T_{x(i+1),y}]}{\frac{L}{N_p}} = \frac{C_{\min}}{N_n} [T_{n(i,y)} - T_{n(i+1),y}] \end{aligned} \quad (8)$$

By the same method the outlet temperatures of the elements on the side of C_{\min} may be obtained using equations (7) and (8).

First row elements

$$T_{n(N+1),y} = F_1 T_{n(N,y)} - F_2 T_{n(N,y)} + F_3 [T_{n(N,y)} + T_{n(N+1),y}] \quad (9)$$

Middle row elements

$$\begin{aligned} T_{n(i+1),y} = F_1 T_{n(i,y)} - F_2 T_{n(i,y)} + F_3 [T_{n(i,y)} + T_{n(i+1),y}] \\ + T_{n(i+1),y} + T_{n(i+1),y} \end{aligned} \quad (10)$$

Last row elements

$$T_{n(f,g)} = F_1 T_{n(f,g-1)} + F_2 T_{n(f,g)} + F_3 [T_{n(f,g)} + T_{n(f,g+1)}] \quad (11)$$

Solving for the fluid outlet temperature for the elements on the side of C_{\min} from equation (7) gives

$$T_{n(f,g)} = F_1 T_{n(f,g)} + F_2 [T_{n(f,g)} + T_{n(f,g+1)}] \quad (12)$$

which holds for all the elements on the side of C_{\min} . For the values of the constants see Appendix 1.

In addition to the four dimensionless groupings for no conduction,

C_{\min}/C_{\max} = capacity rate ratio of fluid flow streams

C_r/C_{\min} = capacity rate ratio of rotor matrix to minimum
fluid capacity rate

$(hA)^* =$ conductance ratio

$NTU_o =$, over-all number of transfer units;

two additional dimensionless groupings are needed in the coefficients of equations (3), (4), (5), (6), (9), (10), (11), and (12) to take conduction into account. These conduction parameters may be defined as:

$\lambda_1 = \frac{hA}{C_{\max} L}$, conduction parameter on side of C_{\max}

$\lambda_2 = \frac{hA}{C_{\min} L}$, conduction parameter on side of C_{\min} .

Also $\frac{\lambda_1}{\lambda_2} = \frac{C_{\min}}{C_{\max}} = \frac{1}{C_{\min}/C_{\max}}$

since the thermal conductivity and the length of the matrix are the same for both sides, and where

$$\lambda = \frac{U}{A_s} \quad , \text{ conduction area ratio.}$$

Another form of defining the conduction parameter is

$$\lambda = \frac{U}{A_s} = \lambda_s \left[1 + \frac{1}{\alpha} \right] \quad , \text{ total conduction parameter}$$

In this case A_s is the total solid area available for conduction, that is

$$A_s = A_{s,x} + A_{s,y}.$$

This form offers the advantage that for a given value of the conduction parameter, the resulting effectiveness will not be affected by small changes in A_s^* .

From the schematic representation shown in Fig. 1b, it should be noted that the left edge of the regenerator is physically the same as the right edge and, therefore, the matrix inlet temperature for the elements of the first column on the side of C_{\max} is the same as the outlet temperature of the corresponding element of the last column on the side of C_{\min} . Expressed mathematically,

$$T_{k(1)} = T_{(k+N_c+1)}$$

where k and f can take the values of 1, 2, 3, N_r .

This is referred to as the reversal condition.

From equations (3), (4), (5), (9), (10), and (11) it is seen that in order to solve for the outlet matrix temperature for a particular element, it is necessary to know the matrix temperatures of the next element. An estimate of these values can be made by determining the

temperature distribution for the case of zero conduction in the direction of fluid flow, since the constants D_3 and D_8 in equations (3), (4) and (5) and the constants F_3 and F_8 in equations (9), (10) and (11) are zero in this case.

A temperature scale can be used for which the fluid entrance temperature is zero at the side of C_{min} and unity at the side of C_{max} .

$$t_n(1,g) = 0 \quad \text{for } g = 1,2,3,\dots,N_n$$

$$t_x(1,j) = 1 \quad \text{for } j = 1,2,3,\dots,N_x$$

For calculation purposes a temperature distribution is assumed on the left edge and the problem is solved for the no conduction case in order to obtain an initial estimate of matrix temperatures for working the problem with conduction. This also provides the no conduction effectiveness necessary for comparison.

The outlet temperatures are calculated for every element by repetitive use of equations (3), (4), (5) and (6) depending on the location of the element. The calculation is started with the first element, first column on the side of C_{max} and then working down the column. When the first column of elements is completed, the second column is calculated and so on until the outlet temperatures of all the elements on the side of C_{max} are determined. It was found that convergence could be enhanced if two passes per column were made before proceeding to the next one. The reason for this is that the equations for the outlet temperatures contain the matrix temperatures of the next element which are necessary to estimate.

At the seal represented by the double line in Fig. 1b,

$$T_{x(1, N_x + 1)} = T_{n(f, 1)}$$

since the matrix outlet temperature for the elements of the last column on the side of C_{\max} is physically the same as the inlet temperature of the corresponding element of the first column on the side of C_{\min} .

The same method is applied to the side of C_{\min} as to the side of C_{\max} . If the temperature distribution assumed on the left edge was correct it would then be duplicated on the right (i.e., the reversal condition is fulfilled). If, however, this is not the case, the resulting temperatures on the right side $(T_{n(f, N_n + 1)})$ are now used on the left $(T_{x(1, 1)})$ and the procedure is repeated. After each pass (i.e., the complete calculation of a temperature distribution) an energy balance is made; and before the solution is accepted for a particular set of parameters, the heat balance error together with the reversal condition has to be fulfilled to the specified accuracy.

The effectiveness of a heat exchanger is defined (Appendix 2) as

$$P = \frac{C_{\max} (t_{x_i} - t_{x_o})}{C_{\min} (t_{x_i} - t_{n_i})} = \frac{t_{n_o} - t_{n_i}}{t_{x_i} - t_{n_i}}.$$

From the conditions of the problem the above expression for effectiveness reduces to

$$e = \left[t_{n(N+1)} \right]_{avg}.$$

The heat balance error was computed from

$$error = 1 - \frac{\sum_{j=1}^N \left[t_{n(N+1+j)} \right]_{avg}}{1 - \left[t_{x(N+j)} \right]_{avg}}.$$

That the solution will converge (i.e., the reversal condition met and the heat balance error converge to zero) will depend on the number of elements used. More important than this is the relation existing between the number of subdivisions for the three streams. The sufficient but not necessary conditions for convergence are (Appendix 1) that

$$N_x > \left(\frac{\lambda_x}{(hA)^*} \right) \frac{1}{C_{\min}} N_r^2$$

$$N_n > \lambda_n \frac{1}{C_{\min}} N_r^2$$

Fig. 2 is a plot of the different values of effectiveness obtained for the following set of parameters:

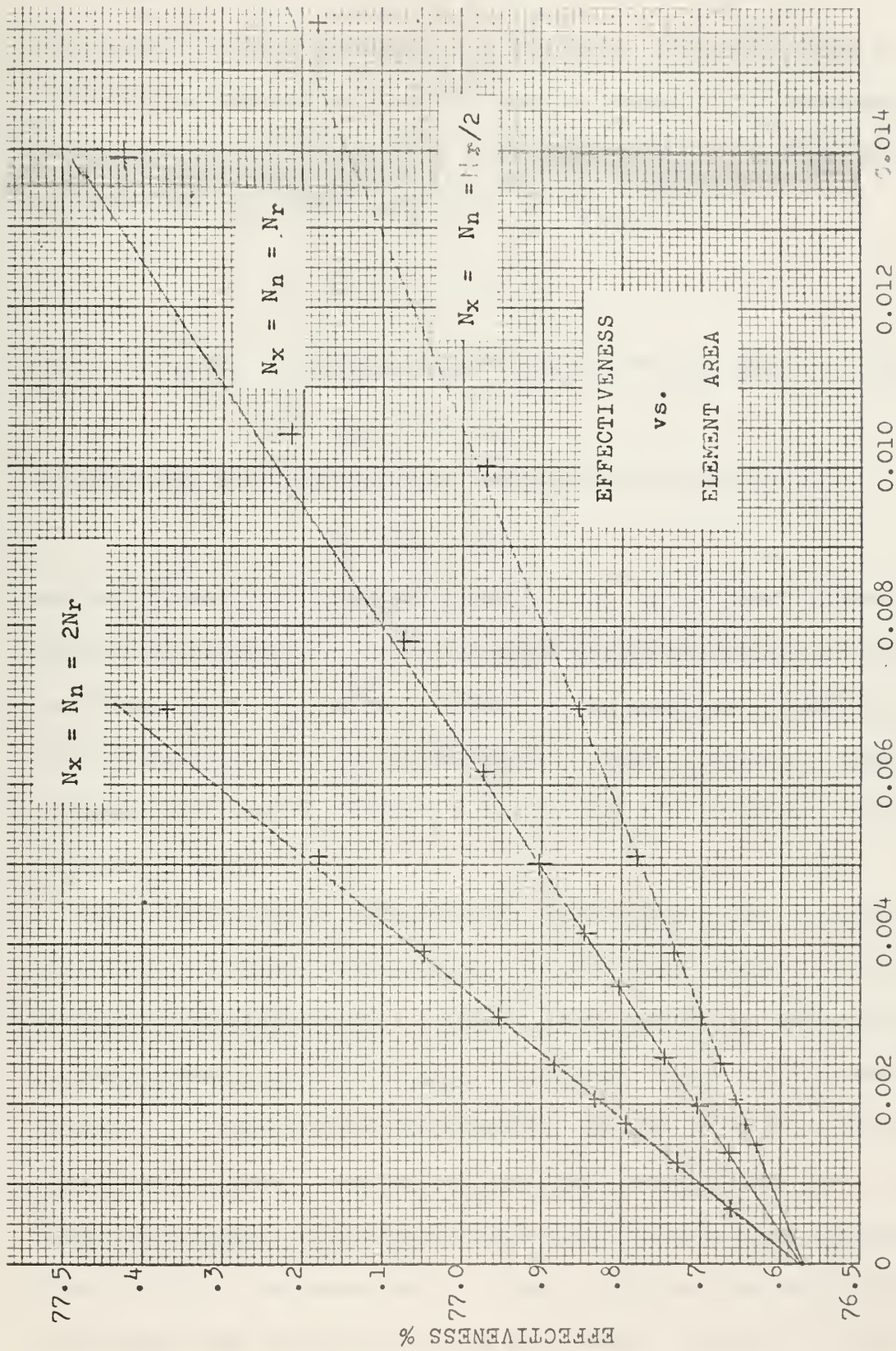
$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 2.0, (hA)^* = 1.0, NTU_0 = 10.0, \lambda = 0.1, As^* = 1.0.$$

Several curves are obtained depending on the relation existing between the number of subdivisions for every stream. To extrapolate the effectiveness to an infinite number of elements, these curves may be approximated by straight lines. The error introduced will depend on the number of elements used and will not be constant for all the range of the parameters. As an illustration consider the set of parameters used to plot Fig. 2. Three cases are investigated:

- 1) $N_x = N_n = N_r/2$
- 2) $N_x = N_n = N_r$
- 3) $N_x = N_n = 2N_r$

A linear extrapolation is performed using the values of effectiveness obtained for values of $\frac{1}{N_r(N_x + N_n)}$. From the results it is found

that the minimum N_r necessary to give an error in the extrapolated effectiveness of less than 0.01% is 18, for which $e_\infty = 76.57\%$



$1/N_r(N_x+N_n)$

Fig 2

for the three cases. The determining factor in the extrapolation is the number of matrix elements. The higher the value of N_T used for the extrapolation the smaller the error in the straight line approximation. Unfortunately, an increase in N_T will bring a higher increase of N_X and N_n , since the convergence condition for this example is

$$N_X = N_n > 0.05 N_T^2.$$

That this is only a sufficient condition can be seen from the case of

$$N_X = N_n = N_T/2$$

for which the solution converges for values of $N_T > 5$; and when

$$N_X = N_n = N_T$$

the solution will still converge for $N_T > 20$. It is true, however, that the closer the number of matrix elements are to the sufficient convergence condition, the fewer passes it takes for the solution to converge to the specified error in the effectiveness. Table 1 shows the number of passes necessary to obtain a solution with a heat balance error of less than 0.005%.

For a given set of parameters the effectiveness was determined first using one set of subdivisions and then recalculated increasing the number of subdivisions. A linear extrapolation was performed to obtain the effectiveness corresponding to an infinite number of elements. The accuracy for each value of effectiveness was specified to be of four significant figures. When these values were extrapolated to an infinite number of elements, the accuracy was reduced to only three significant figures since a compromise had to be made with the time available for computations and the number of runs to be made. However, the method

TABLE 1

Number of passes necessary to obtain an effectiveness with a heat balance error of less than 0.005% for the following set of parameters:

$$C_{\min}/C_{\max} = 1.0, \quad C_r/C_{\min} = 2.0, \quad (hA)^* = 1.0,$$

$$NTU_0 = 10.0, \quad \lambda = 0.1, \quad As^* = 1.0$$

N_r	N_x	N_n	No. of passes	$e_{\infty}^* \%$
12	12	12	27	76.581
12	24	24	22	76.581
14	7	7	41	76.578
14	14	14	34	76.576
14	28	28	27	76.576
16	16	16	41	76.574
16	32	32	32	76.574
18	9	9	62	76.573
18	18	18	50	76.572
18	36	36	38	76.572
20	20	20	60	76.572
22	22	22	71	76.571

* e_{∞} is obtained by a linear extrapolation of the effectiveness for N_r and N_r-1 .

imposes no restriction as to the ultimate accuracy of the solution.

When working with very high values of C_r/C_{min} the computation time becomes considerably long. This is due to the fact that the final temperatures differ very little from the assumed initial temperatures and a large number of elements is required. The starting estimates of matrix temperatures then become very important. For the limiting case of C_r/C_{min} equal to infinite, there is no temperature variation in the direction of metal flow so that the method without modifications is not applicable. For this case, however, the exact solution of Hahnemann [3] is available.

The Fortran system was used to program the above method for the CDC 1604 digital computer. The matrix notation employed in the equations derived previously is very convenient in writing up the program, since several loops can be made by just changing the subscripts of the variables involved. The complete computer program is given in Appendix 2.

3. Discussion of results

The solutions generated are presented in tabular form in Tables 2 through 25 and graphically in Figs. 3 and 4. The accuracy in the effectiveness was specified to be of three significant figures. It was found that for

$$0.7 (hA)^* \leq As^* \leq 1.3 (hA)^*$$

the resulting values of effectiveness were almost identical to the ones calculated for

$$As^* = (hA)^*$$

so that these were the values used in preparing the Tables.

By using NTU_0 and λ as defined in the nomenclature, which contain $(hA)^*$ and As^* explicitly, the influence of these two parameters on the conduction effect, $\frac{\Delta e}{e}$, were found to be very small for values of $C_{min}/C_{max} \geq 0.9$ and $\lambda \leq 0.1$, which is the range for the gas turbine regenerator problem.

Figs. 3 and 4 show the effect of conduction on effectiveness for values of $C_{min}/C_{max} = 1.0$ and 0.9 respectively, with different values of NTU_0 , C_r/C_{min} and λ , up to a limit of 10%. The conduction effect continues to increase with increasing λ . For small values of λ this effect increases nearly exponentially. By further increasing λ , a saturation is reached which approaches the limiting condition of $\lambda = \infty$. Fig. 5 is a representative curve which shows the limiting influence on the conduction effect by continuously increasing λ for a given set of parameters.

The conduction effect for values of $C_r/C_{min} = 10.0$ are almost identical with the values reported by Schultz [2] and Hahnemann [3], [6] for the limiting case of $C_r/C_{min} = \infty$.

For small values of NTUo the effect of increasing C_r/C_{\min} is to increase slightly the conduction effect, as can be seen from Fig. 6. For higher values of NTUo the effect of conduction is reduced with increasing C_r/C_{\min} .

The effect of increasing NTUo upon $\frac{\Delta e}{e}$ is intimately related to the value of C_r/C_{\min} and C_{\min}/C_{\max} . This effect can best be explained by observing Fig. 6. For the case where $C_{\min}/C_{\max} = 1.0$ the effect of increasing NTUo will result in an increase in $\frac{\Delta e}{e}$ regardless of the value of C_r/C_{\min} ; of course, the higher the value of C_r/C_{\min} the smaller this effect will be for relatively high values of NTUo. Now, by reducing C_{\min}/C_{\max} and increasing C_r/C_{\min} a point is reached in which an increase in the NTUo will result in a reduction in $\frac{\Delta e}{e}$. The point where this occurs will depend upon the value of the conduction parameter λ .

4. Summary and Conclusions

A finite difference numerical analysis is presented which will determine the effect of the longitudinal heat conduction in rotary regenerators for steady state conditions.

The ranges of governing parameters which have been covered are:

$$0.5 \leq C_{\min}/C_{\max} \leq 1.0$$

$$1.0 \leq C_r/C_{\min} \leq 10.0$$

$$1.0 \leq NTU_0 \leq 20.0$$

$$0.25 \leq (hA)^* \leq 1.0$$

$$0 \leq \lambda \leq 0.2$$

$$0.15 \leq As^* \leq 1.3$$

The ranges of parameters of interest for gas turbine regenerator application are plotted in Figs. 3 and 4. A more accurate determination of the conduction effect than that given in the figures can be made by interpolating values from the Tables.

The computer program used to carry out the calculations on the CDC 1604 digital computer is included in Appendix 2. This program imposes no restrictions on the parameters for which the analysis would be applicable. It is recommended, however, that for values of $C_r/C_{\min} > 10$ the solution for $C_r/C_{\min} = 10$ be used. For this case the computer time for a solution increases considerably and the error introduced by assuming $C_r/C_{\min} = 10$ is negligible.

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TABLE 2 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 1.0, (hA)^* = As^* = 1.0$$

NTU ₀ \ λ	e %	Conduction effect, $\frac{\Delta e}{e}$ %							
	0	0.01	0.02	0.04	0.08	0.14	0.16	0.20	0.32
1	46.64	0.42	0.78	1.48	2.61	3.94	4.30	4.95	6.42
2	60.06	0.58	1.11	2.10	3.81	5.93	6.54	7.65	10.03
3	66.72	0.68	1.32	2.50	4.58	7.17	7.92	9.30	12.60
4	70.86	0.77	1.48	2.82	5.17	8.08	8.93	10.48	14.22
5	73.75	0.84	1.63	3.09	5.65	8.81	9.73	11.41	15.44
6	75.92		1.76		6.05	9.42		12.16	
7	77.63		1.87		6.41	9.93		12.80	
8	79.01		1.97		6.72	10.38		13.34	
9	80.16		2.07		7.01	10.78		13.82	
10	81.15	1.12	2.16	4.05	7.26	11.13	12.23	14.24	18.97
12	82.74		2.32		7.71	11.74		14.95	
14	84.00		2.46		8.10	12.25		15.54	
16	85.01		2.60		8.44	12.69		16.04	
18	85.86		2.72		8.74	13.08		16.47	
20	86.58	1.49	2.83	5.17	9.01	13.42	14.60	16.85	21.96
40	90.37	1.92	3.55	6.30	10.54		16.58		24.25
100	93.87	2.70	4.78	7.97	12.55		18.79		26.54
500	97.87	4.04	6.80	10.57	15.42		21.65		29.27
1000	98.81	4.39	7.30	11.17	16.07		22.30		29.88

TABLE 3 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 1.2, (hA)^* = As^* = 1.0$$

NTU ₀ \ λ	e %	Conduction effect, $\frac{U_c}{U} \%$			
	0	0.02	0.08	0.14	0.20
1	47.62	0.81	2.66	4.01	5.04
2	61.97	1.13	3.88	6.05	7.81
3	69.15	1.33	4.64	7.28	9.46
4	73.61	1.49	5.21	8.18	10.63
5	76.72	1.62	5.66	8.89	11.54
6	79.04	1.73	6.05	9.47	12.28
7	80.85	1.84	6.38	9.96	12.89
8	82.31	1.93	6.67	10.38	13.40
9	83.53	2.01	6.93	10.76	13.86
10	84.55	2.09	7.16	11.08	14.26
11	85.44	2.16	7.37	11.38	14.61
12	86.21	2.22	7.56	11.65	14.93
13	86.89	2.28	7.74	11.89	15.22
14	87.49	2.34	7.90	12.11	15.48
15	88.03	2.40	8.05	12.31	15.71
16	88.52	2.45	8.19	12.50	15.93
17	88.96	2.50	8.32	12.67	16.13
18	89.37	2.54	8.44	12.83	16.31
19	89.74	2.59	8.56	13.00	16.48
20	90.09	2.63	8.66	13.12	16.64

TABLE 4 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, \quad C_r/C_{\min} = 1.4, \quad (hA)^* = As^* = 1.0$$

NTU _o \ λ	ϵ %	Conduction effect, $\frac{\Delta \epsilon}{\epsilon}$ %							
		.01	.02	.04	.08	.14	.16	.2	.32
1	48.23		.82		2.69	4.05		5.10	
2	63.16		1.14		3.93	6.12		7.90	
3	70.65		1.33		4.67	7.33		9.53	
4	75.27		1.48		5.20	8.20		10.67	
5	78.45		1.60		5.63	8.86		11.54	
6	80.82		1.70		5.98	9.40		12.23	
7	82.65		1.79		6.28	9.85		12.80	
8	84.12		1.87		6.54	10.24		13.28	
9	85.33		1.94		6.76	10.57		13.69	
10	86.34		2.00		6.96	10.86		14.05	
12	87.96		2.12		7.31	11.36		14.64	
14	89.20		2.22		7.59	11.75		15.12	
16	90.18		2.3		7.83	12.08		15.51	
18	90.98		2.37		8.04	12.36		15.84	
20	91.65		2.44		8.22	12.61		16.12	

TABLE 5 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 2.0, (hA)^* = As^* = 1.0$$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %							
NTUo λ	0	.01	.02	.04	.08	.14	.16	.2	.32
1	49.12		.84		2.73	4.11		5.17	
2	64.91		1.16		4.0	6.22		8.04	
3	72.80		1.34		4.70	7.40		9.37	
4	77.60		1.47		5.18	8.19		10.70	
5	80.86		1.56		5.54	8.78		11.47	
6	83.22		1.64		5.83	9.23		12.07	
7	85.03		1.71		6.07	9.61		12.55	
8	86.46		1.76		6.27	9.92		12.95	
9	87.63		1.82		6.44	10.18		13.28	
10	88.59		1.86		6.60	10.41		13.57	
12	90.11		1.94		6.85	10.79		14.04	
14	91.25		2.01		7.05	11.08		14.41	
16	92.14		2.06		7.22	11.33		14.71	
18	92.86		2.11		7.36	11.53		14.95	
20	93.45		2.15		7.49	11.71		15.16	
500	99.65	1.40	2.72	5.11	9.10		15.18		23.40
1000	99.80	1.40	2.72	5.13	9.14		15.24		23.47

TABLE 6 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 10.0, (hA)^* = \Delta s^* = 1.0$$

NTUo	e %	Conduction effect, $\frac{\Delta C}{C} \%$			
	0	.02	.08	.14	.2
1	49.96	.85	2.77	4.16	5.24
2	66.59	1.18	4.04	6.29	8.12
3	74.91	1.35	4.72	7.41	9.63
4	79.89	1.46	5.13	8.10	10.57
5	83.22	1.53	5.42	8.57	11.21
6	85.59	1.58	5.62	8.92	11.67
7	87.38	1.62	5.78	9.19	12.03
8	88.76	1.65	5.91	9.40	12.31
9	89.87	1.68	6.02	9.57	12.54
10	90.78	1.70	6.10	9.71	12.73
12	92.18	1.73	6.24	9.94	13.03
14	93.20	1.76	6.34	10.11	13.26
16	93.99	1.76	6.43	10.25	13.45
18	94.61	1.80	6.49	10.36	13.59
20	95.11	1.81	6.55	10.45	13.71

TABLE 7 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 1.0, (hA)^* = As^* = 0.5$$

NTU ₀ \ λ	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
	0	0.015	0.03	0.06	0.12	0.24	0.48
1	46.62	.62	1.17	2.12	3.64	5.72	8.05
2	60.08	.85	1.64	3.04	5.38	8.82	13.03
3	66.75	1.01	1.95	3.63	6.46	10.67	15.90
4	70.90	1.14	2.19	4.09	7.26	11.98	17.83
5	73.79	1.25	2.40	4.47	7.91	12.98	19.26
10	81.19	1.66	3.15	5.77	9.98	15.98	23.2
20	86.62	2.19	4.07	7.22	12.05	18.65	26.36
40	90.46	2.69	4.92	8.51	13.77	20.73	28.69
100	93.61	3.24	5.84	9.85	15.44	22.60	30.64
500	95.57	3.51	6.36	10.66	16.48	23.76	31.91
1000	95.83	3.55	6.43	10.77	16.62	23.92	32.10

TABLE 8 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\text{min}}/C_{\text{max}} = 1.0, \quad C_r/C_{\text{min}} = 2.0, \quad (hA)^* = (As)^* = .5$$

NTU _o λ	e %	Conduction effect, %						
	0	.015	.03	.06	.12	.21	.24	.3
1	49.11		1.22		3.79	5.51		6.75
2	64.92		1.70		5.62	8.47		10.64
3	72.81		1.97		6.46	10.10		12.78
4	77.61		2.16		7.32	11.17		14.17
5	80.86		2.30		7.82	11.95		15.16
6	83.23		2.42		8.22	12.55		15.92
7	85.04		2.51		8.54	13.03		16.52
8	86.47		2.60		8.81	13.43		17.01
9	87.63		2.67		9.06	13.76		17.42
10	88.59		2.73		9.24	14.05		17.76
12	90.11		2.84		9.57	14.52		18.32
14	91.25		2.94		9.83	14.89		18.76
16	92.14		3.01		10.05	15.18		19.11
18	92.86		3.08		10.23	15.43		19.40
20	93.45		3.14		10.38	15.63		19.64
500	99.60	2.12	3.95	7.15	12.29		19.63	28.54
1000	99.79	2.17	4.0	7.21	12.37		19.73	28.67

TABLE 9 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\text{tain}} = 10.0, (hA)^* = As^* = .5$$

	e %	Conduction effect, $\frac{10}{e}$ %					
NTU _o \ λ	0	.015	.03	.06	.12	.24	.48
1	49.96	0.65	1.23	2.24	3.84	6.04	8.51
2	66.59	0.90	1.72	3.20	5.67	9.32	13.84
3	74.91	1.02	1.97	3.70	6.62	11.04	16.64
4	79.89	1.10	2.12	4.00	7.21	12.09	18.34
5	83.22	1.15	2.23	4.21	7.61	12.80	19.48
10	90.78	1.28	2.49	4.73	8.59	14.51	22.13
20	95.11	1.36	2.65	5.05	9.20	15.58	23.76

TABLE 10 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 1.0, (hA)^* = As^* = .25$$

NTU _o \ λ	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
	0	.025	.05	.1	.2	.4	.8
1	46.59	1.02	1.90	3.38	5.57	8.25	10.84
2	60.13	1.43	2.69	4.88	8.22	12.50	16.84
3	66.84	1.70	3.21	5.80	9.78	14.88	20.07
4	70.10	1.92	3.61	6.50	10.88	16.47	22.16
5	73.90	2.10	3.94	7.05	11.73	17.65	23.66
10	81.28	2.74	5.05	8.81	14.25	20.93	27.63
20	86.64	3.52	6.27	10.56	16.50	23.61	30.62

TABLE 11 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 1.0, C_r/C_{\min} = 10.0, (h\Delta)^* = As^* = .25$$

NTU _o	e %	Conduction effect, $\frac{\Delta p}{\Delta p_0}$ %					
	0	.025	.05	.1	.2	.4	.8
1	49.96	1.07	1.99	7.92	5.84	8.68	11.49
2	66.60	1.47	2.79	5.06	8.59	13.21	18.05
3	74.91	1.68	3.20	5.84	10.00	15.53	21.43
4	79.89	1.80	3.44	6.32	10.86	16.93	23.48
5	83.22	1.88	3.61	6.64	11.44	17.88	24.86
10	90.78	2.1	4.03	7.44	12.83	20.10	28.02
20	95.11	2.23	4.29	7.92	13.69	21.46	29.83

TABLE 12 LONGITUDINAL HEAT CONDUCTION EFFECT

$$q_{\text{air}}/q_{\text{max}} = .9, \quad q_T/q_{\text{min}} = 1.0, \quad (RA)^* = Ag^* = 1.0$$

NTVO	ϕ %	Conduction effect, $\frac{\Delta q}{q}$ %					
	0	.01	.02	.04	.08	.16	.32
1	47.63	0.38	0.73	1.36	2.41	3.98	5.95
2	61.56	0.53	1.02	1.93	3.52	6.08	9.62
3	68.46	0.62	1.21	2.30	4.24	7.37	11.80
4	72.74	0.70	1.36	2.60	4.78	8.32	13.33
5	75.72	0.76	1.49	2.84	5.22	9.06	14.50
10	83.29	1.01	1.96	3.70	6.70	11.39	17.85
20	88.69	1.32	2.52	4.66	8.21	13.50	20.60
40	92.41	1.68	3.15	5.64	9.58	15.32	22.74
100	95.50	2.27	4.08	6.94	11.17	17.10	24.67

TABLE 13 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = .9, C_r/C_{\min} = 2.0, (hA)^* = As^* = 1.0$$

	ϵ %	Conduction effect, $\frac{\Delta}{\Delta_0}$ %					
NTU ₀	0	.01	.02	.04	.08	.16	.32
1	50.30	0.40	0.77	1.43	2.53	4.17	6.23
2	66.92	0.55	1.07	2.03	3.71	6.41	10.16
3	75.30	0.63	1.24	2.37	4.37	7.66	12.36
4	80.40	0.69	1.35	2.59	4.82	8.50	13.80
5	83.85	0.73	1.43	2.76	5.14	9.10	14.82
10	91.99	0.85	1.67	3.23	6.04	10.71	17.44
20	96.82	0.90	1.79	3.49	6.59	11.74	19.11

TABLE 14 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = .9, C_r/C_{\min} = 10.0, (hA)^* = As^* = 1.0$$

NTU _o λ	e %	Conduction effect, $\frac{1-e}{\sigma}$ %					
	0	.01	.02	.04	.08	.16	.32
1	51.22	0.41	0.78	1.45	2.56	4.23	6.32
2	68.81	0.56	1.09	2.06	3.77	6.49	10.28
3	77.67	0.63	1.24	2.38	4.39	7.68	12.38
4	82.98	0.68	1.33	2.56	4.76	8.39	13.65
5	86.52	0.70	1.39	2.68	5.00	8.87	14.49
10	94.36	0.75	1.48	2.88	5.48	9.87	16.35
20	98.36	0.65	1.33	2.70	5.35	10.03	17.08

TABLE 15 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = .9, C_r/C_{\min} = 1.0, (hA)^* = As^* = .5$$

NTU _o	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
	0	.015	.03	.06	.12	.24	.48
1	47.53	0.54	1.03	1.88	3.23	5.11	7.23
2	61.40	0.74	1.43	2.67	4.76	7.89	11.81
3	68.26	0.87	1.69	3.17	5.70	9.55	14.48
4	72.51	0.97	1.88	3.55	6.40	10.73	16.29
5	75.47	1.06	2.05	3.86	6.94	11.63	17.64
10	82.98	1.35	2.61	4.89	8.69	14.33	21.38
20	88.34	1.68	3.23	5.97	10.39	16.70	24.35

TABLE 16 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = .9, \quad C_r/C_{\min} = 2.0, \quad (hA)^* = As^* = .5$$

NTU _o	e %	Conduction effect, $\frac{\Delta e}{e}$ %			
	0	.03	.12	.21	.30
1	50.27	1.03	3.40	4.96	6.09
2	66.88	1.53	5.08	7.71	9.73
3	75.23	1.77	6.01	9.23	11.76
4	80.32	1.93	6.63	10.24	13.08
5	83.77	2.05	7.09	10.97	14.03
6	86.28	2.14	7.45	11.53	14.75
7	88.18	2.22	7.74	11.93	15.32
8	89.69	2.29	7.97	12.34	15.78
9	90.90	2.34	8.17	12.64	16.17
10	91.91	2.38	8.34	12.90	16.49
12	93.47	2.45	8.60	13.31	17.00
14	94.62	2.50	8.80	13.61	17.39
16	95.50	2.54	8.96	13.85	17.67
18	96.20	2.56	9.07	14.03	17.90
20	96.76	2.57	9.16	14.18	18.09

TABLE 17 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\text{MIN}}/C_{\text{MAX}} = .9, C_1/C_{\text{MAX}} = 10.0, (hA)^* = As^* = .5$$

	ϵ %	Conduction effect, $\frac{\Delta T}{T_b} - \%$					
NTU ₀	0	0.015	0.03	0.06	0.12	0.24	0.48
1	51.22	0.58	1.10	2.01	3.45	5.45	7.73
2	63.80	0.80	1.55	2.99	5.15	8.55	12.83
3	77.66	0.92	1.78	3.35	6.04	10.19	15.56
4	82.98	0.98	1.91	3.62	6.58	11.18	17.22
5	86.51	1.02	1.99	3.79	6.39	11.85	18.32
10	94.36	1.07	2.12	4.11	7.65	13.29	20.77
20	98.35	0.96	1.96	3.97	7.71	13.80	21.91

TABLE 18 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = .9, C_r/C_{\min} = 1.0, (NA)^* = As^* = .25$$

NTU ₀	e %	Conduction effect, $\frac{\Delta T}{T} \%$					
	0	0.025	0.05	0.1	0.2	0.4	0.8
1	47.40	0.07	1.62	2.90	4.82	7.21	9.57
2	61.29	1.18	2.26	4.13	7.08	10.98	15.09
3	68.13	1.39	2.65	4.87	8.39	13.10	18.12
4	72.37	1.54	2.94	5.41	9.32	14.54	20.11
5	75.31	1.66	3.17	5.83	10.03	15.61	21.54
10	82.74	2.05	3.92	7.17	12.16	18.61	25.35
20	88.02	2.43	4.67	8.45	14.04	21.04	28.16

TABLE 19 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\text{min}}/C_{\text{max}} = .2, \quad C_F/C_{\text{min}} = 10.0, \quad (\Delta t)^* = \Delta s^* = .25$$

NTU λ	e %	Conduction effect, $\frac{\Delta t}{t} \%$					
	0	0.00	0.05	0.1	0.2	0.4	0.5
1	51.22	0.43	1.74	3.10	5.15	7.74	10.36
2	68.80	1.29	2.45	4.49	7.72	12.06	16.75
3	77.66	1.47	2.82	5.21	9.06	14.32	20.12
4	82.98	1.58	3.03	5.64	9.37	15.70	22.10
5	86.51	1.64	3.17	5.93	10.41	16.62	23.57
10	94.36	1.74	3.41	6.48	11.56	18.67	26.62
20	98.35	1.59	3.26	6.76	11.89	19.54	28.00

TABLE 20 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 0.5, \quad C_r/C_{\min} = 1.0, \quad (\ln A)^* = \Delta s^* = 1.0$$

NTU ₀	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
	0	0.01	0.02	0.04	0.08	0.16	0.32
2	66.89	0.35	0.68	1.29	2.37	4.15	6.71
3	74.01	0.41	0.80	1.53	2.84	5.02	8.25
4	78.16	0.45	0.89	1.71	3.19	5.65	9.29
5	80.89	0.49	0.97	1.86	3.46	6.12	10.06
10	87.15	0.65	1.26	2.39	4.38	7.57	12.15
20	91.07	0.86	1.64	3.04	5.37	8.94	13.87

TABLE 21 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 0.5, C_r/C_{\min} = 10.0, (hA)^* = As^* = 1.0$$

	e %	Conduction effect, $\frac{hA}{C_r}$ %					
NTU _o	0	0.01	0.02	0.04	0.08	0.10	0.32
1	56.42	0.28	0.53	0.98	1.74	2.90	4.37
2	77.34	0.37	0.71	1.35	2.49	4.36	7.06
3	87.30	0.38	0.74	1.43	2.68	4.82	8.03
4	92.60	0.35	0.70	1.36	2.60	4.80	8.29
5	95.59	0.31	0.62	1.22	2.39	4.53	8.1
10	99.62	0.10	0.22	.49	1.14	2.68	5.96
20	99.99	0.01	0.01	0.05	0.23	1.02	3.65

TABLE 21. CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 0.5, C_r/C_{\min} = 1.0, (kA)^* = As^* = 0.5$$

NTDS	ϵ %	Conduction effect, $\frac{\Delta t}{t}$ %					
	ϵ	0.01	0.02	0.04	0.08	0.16	0.32
1	50.97	0.28	.53	0.98	1.71	2.75	3.99
2	65.65	0.35	.69	1.32	2.40	4.15	6.56
3	72.44	0.39	.78	1.49	2.78	4.92	8.04
4	76.43	0.42	.87	1.62	3.04	5.46	9.04
5	79.10	0.45	0.89	1.72	3.25	5.86	9.79
10	85.44	0.56	1.09	2.12	4.09	7.8	11.95
20	89.72	0.74	1.44	2.72	4.95	8.58	13.90

TABLE 22. TRANSDUCER HEAT CONDUCTION EFFECT

$$Q_{\text{in}}/Q_{\text{out}} = 1.5, Q_{\text{in}}/Q_{\text{out}} = 10.5, (hA)^2 = 1.5^2 = 0.5$$

	ϵ %	Conduction effect, $\frac{\Delta T}{T}$ %					
ΔT	0	0.01	0.02	0.04	0.08	0.16	0.32
1	52.41	0.77	0.61	1.01	1.95	3.14	4.54
2	77.32	0.43	0.33	1.57	2.87	4.94	7.76
3	87.27	0.44	0.37	1.66	3.14	5.61	9.12
4	92.57	0.42	0.32	1.62	3.10	5.71	9.73
5	95.56	0.37	0.74	1.47	2.90	5.53	9.76
10	99.61	0.13	0.22	0.53	1.54	3.74	8.13
20	99.99	0.01	0.02	0.08	0.43	1.92	6.06

TABLE 24 LONGITUDINAL HEAT CONDUCTION EFFECT

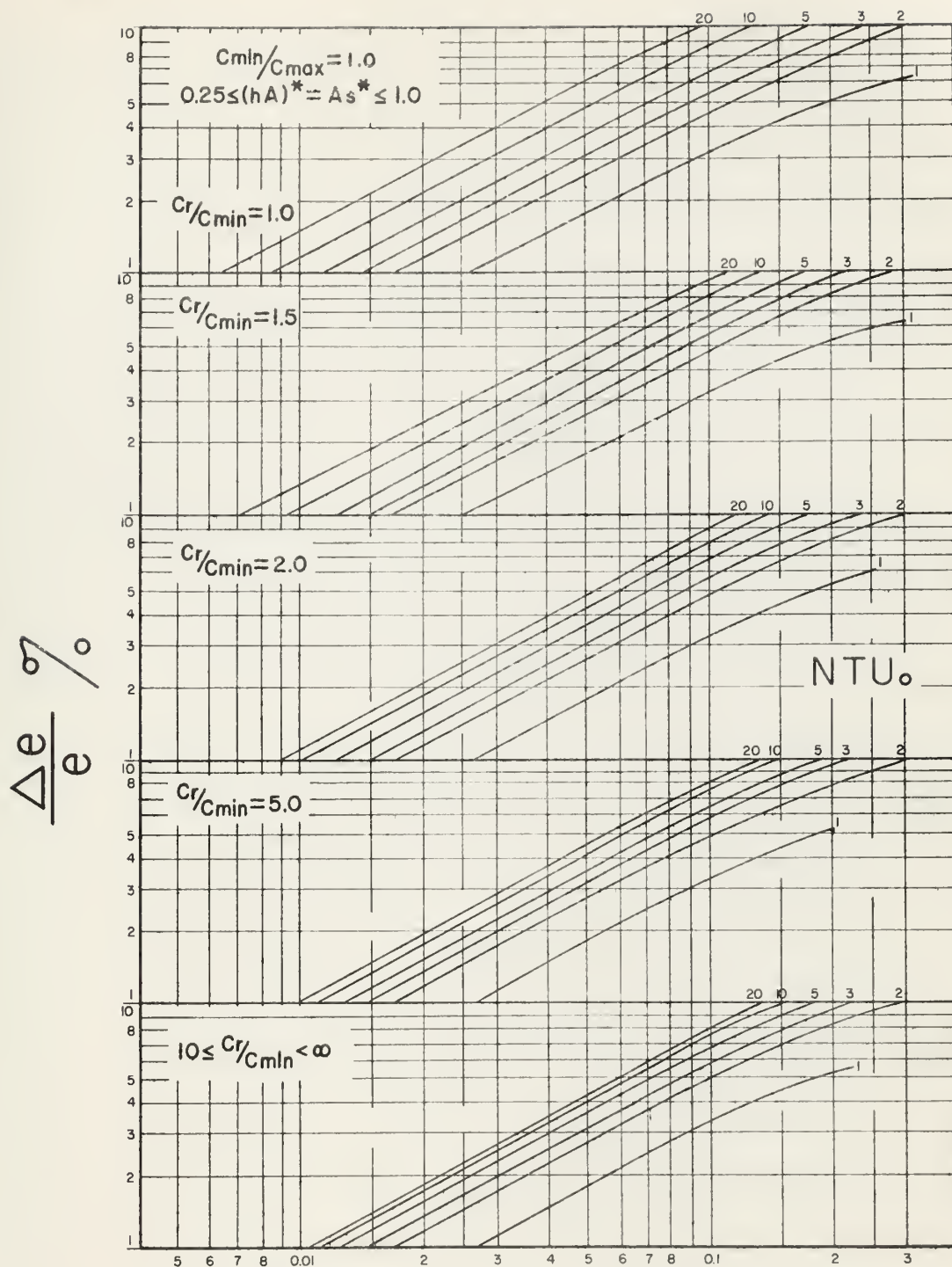
$$C_{\min}/C_{\max} = 0.5, C_r/C_{\min} = 1.0, (hA)^* = As^* = 0.25$$

NTU _o	e %	Conduction effect, $\frac{e}{e_0}$ %					
	0	.01	.02	.04	.08	.16	.32
1	50.21	0.34	0.65	1.13	2.01	3.12	4.54
2	64.39	0.38	0.75	1.42	2.61	4.47	6.89
3	70.98	0.39	0.77	1.51	2.89	5.19	8.38
4	74.91	0.39	0.79	1.58	3.09	5.70	9.43
5	77.57	0.40	0.81	1.64	3.26	6.11	10.24
10	84.12	0.49	0.99	1.99	3.97	7.53	12.71
20	88.72	0.66	1.30	2.54	4.92	9.07	14.97

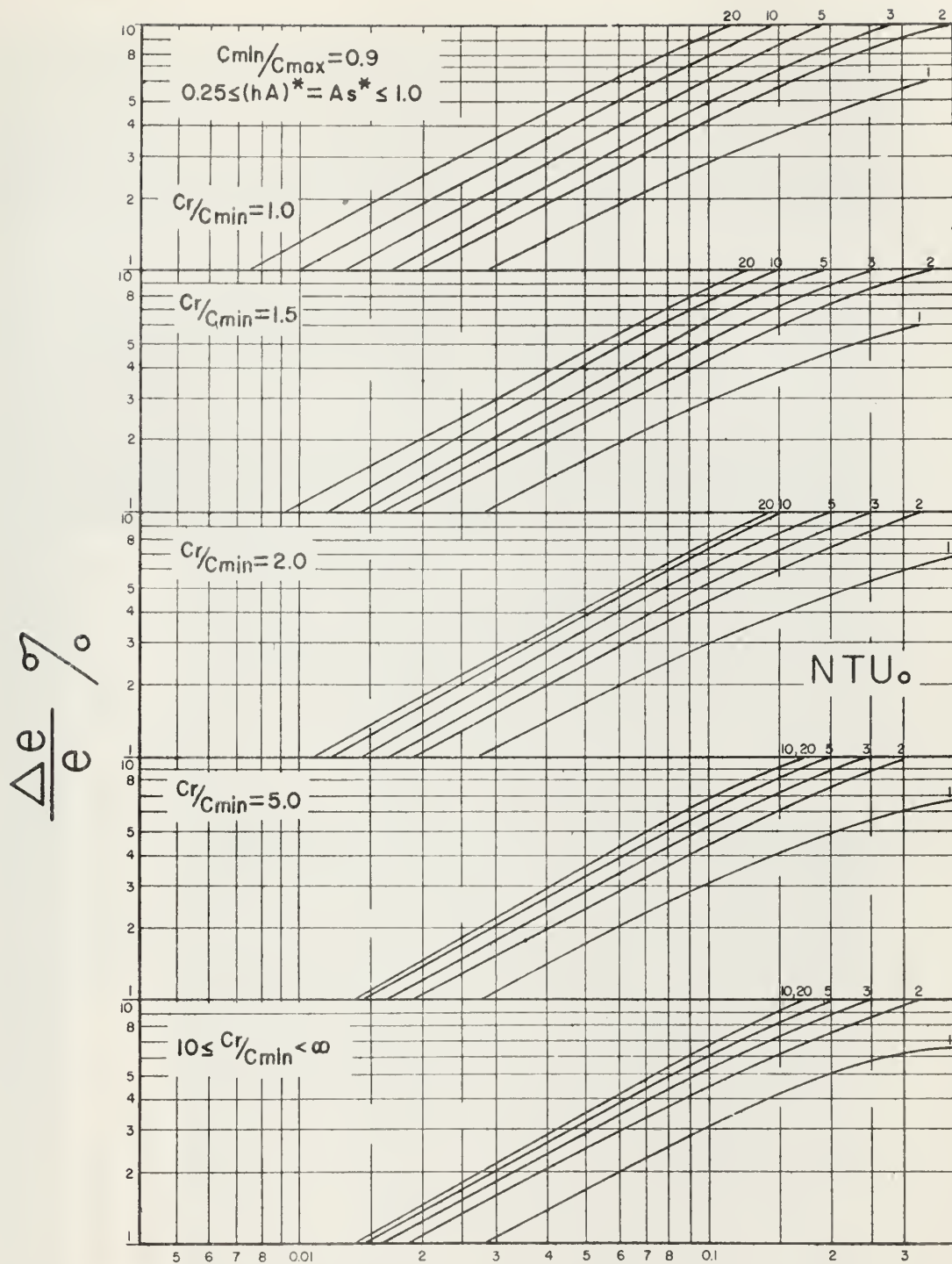
TABLE 25 LONGITUDINAL HEAT CONDUCTION EFFECT

$$C_{\min}/C_{\max} = 0.5, C_r/C_{\min} = 10.0, (hA)^* = As^* = 0.25$$

NTU _o	e %	Conduction effect, — %					
	0	.01	.02	.04	.08	.16	.32
1	56.40	0.43	0.81	1.47	2.51	3.92	5.46
2	77.30	0.58	1.11	2.10	3.79	6.36	9.59
3	87.24	0.60	1.18	2.27	4.24	7.42	11.66
4	92.53	0.57	1.12	2.22	4.28	7.77	12.62
5	95.53	0.50	1.02	2.05	4.10	7.73	12.99
10	99.60	0.18	0.41	1.01	2.59	6.18	12.19
20	99.99	0.01	0.04	0.22	1.13	4.29	10.59



λ
 FIG. 3



λ
 FIG. 4

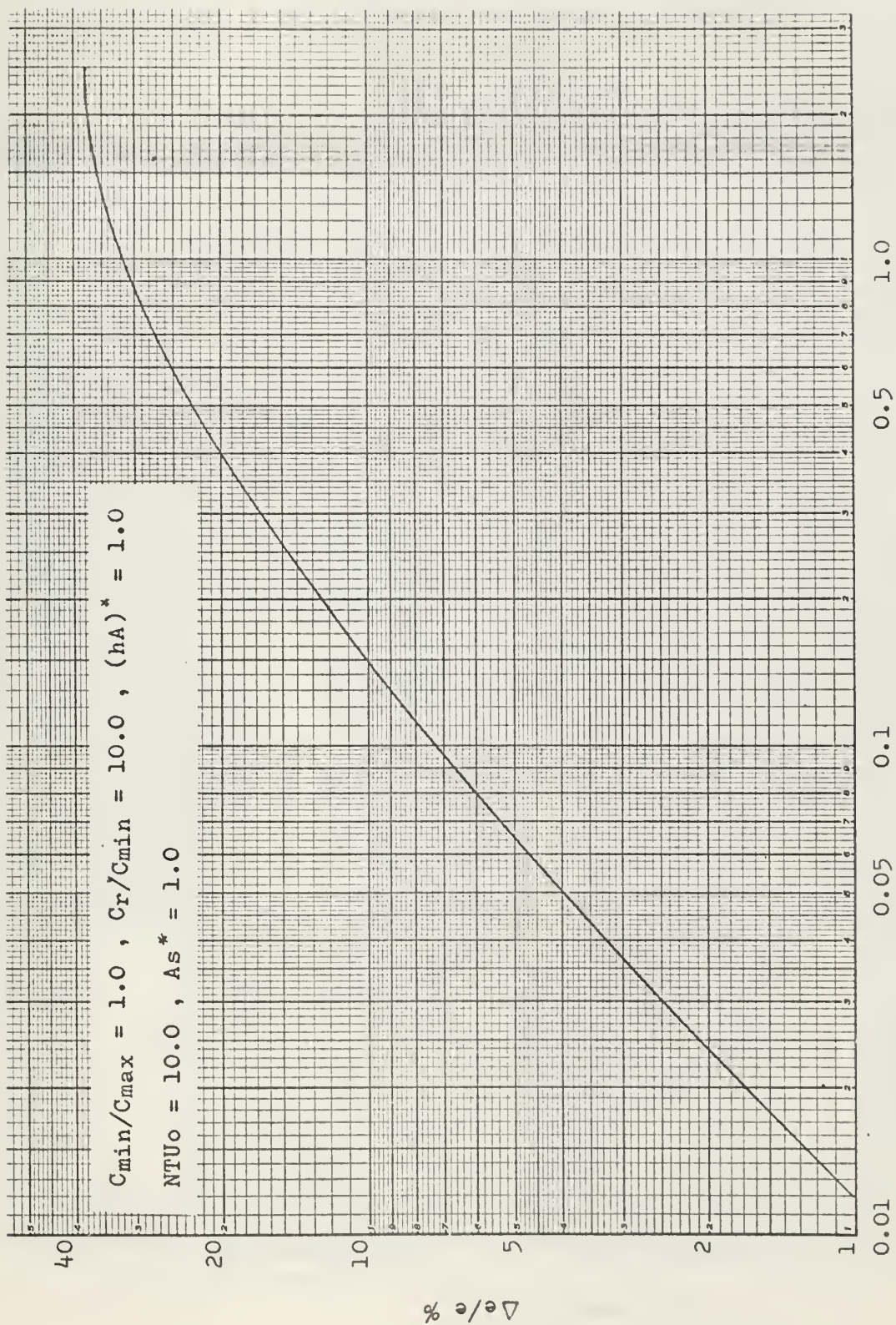
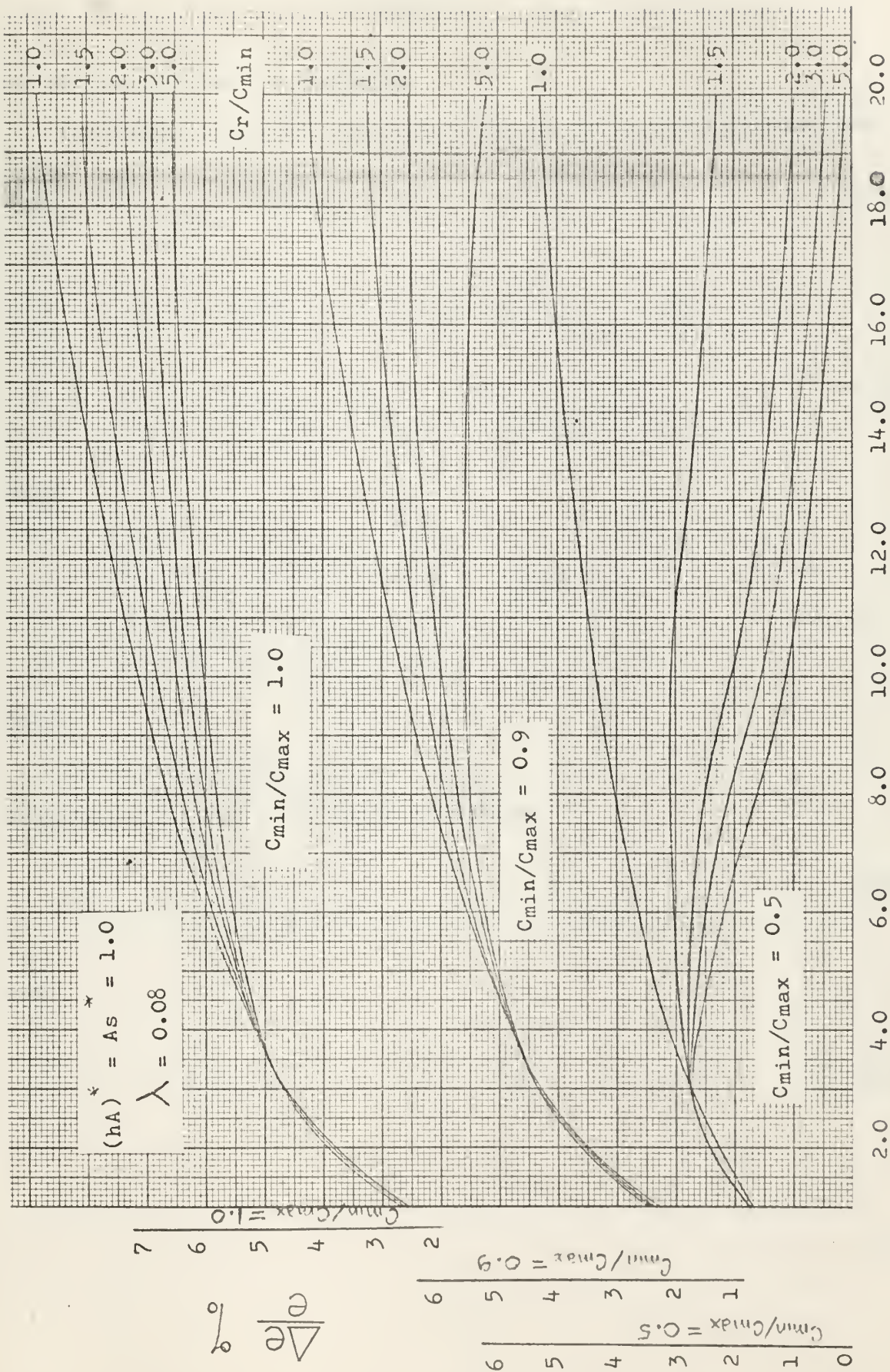


Fig. 5



NTU_o

Fig. 6

APPENDIX 1.

Derivation of iterative equations.-

Equations (1) and (2) from page 5 can be put in the following form:

$$t_{x(i,j)} - t_{x(i+1,j)} = C_1 [T_{x(i,j)} + T_{x(i+1,j)} - T_{x(i,j)} - T_{x(i,j+1)}] \quad (1.1)$$

$$t_{x(i,j)} - t_{x(i+1,j)} = C_2 [T_{x(i,j)} - T_{x(i,j)}] - C_3 [T_{x(i-1,j)} + T_{x(i-1,j+1)} - 2(T_{x(i,j)} + T_{x(i+1,j)}) + T_{x(i-1,j)} + T_{x(i+1,j+1)}] \quad (1.2)$$

where

$$C_1 = \frac{1}{h} \frac{1}{\rho c_p} = \frac{h}{\rho c_p} \frac{1}{h} \frac{1}{\rho c_p} \left[1 + \frac{1}{h} \right] \frac{1}{2k}$$

$$C_2 = \frac{1}{h} \frac{1}{\rho c_p} = \frac{h}{\rho c_p} \frac{1}{h} \frac{1}{\rho c_p} \frac{1}{2k}$$

and

$$C_3 = \frac{1}{h} \frac{1}{\rho c_p} = \frac{h}{\rho c_p} \frac{1}{h} \frac{1}{\rho c_p} \frac{1}{2k}$$

Equations (1.1) and (1.2) may be written as

$$t_{x(i,j)} - t_{x(i+1,j)} = (C_1 + C_2) T_{x(i,j)} - C_1 T_{x(i,j+1)} - C_2 T_{x(i-1,j)} - C_3 T_{x(i-1,j+1)} - C_3 T_{x(i+1,j+1)} \quad (1.3)$$

and

$$-t_{x(i,j)} - (C_1 + C_2) T_{x(i,j)} = -T_{x(i,j)} + (2C_1 - C_2) T_{x(i,j)} - C_1 T_{x(i,j+1)} - C_2 T_{x(i-1,j)} - C_3 T_{x(i-1,j+1)} - C_3 T_{x(i+1,j+1)} \quad (1.4)$$

By adding to equation (1.3) the product of $(1+C_1)$ (1.4) and rearranging

$$T_{n+1} = D_1 T_n + D_2 [T_{n-1} + T_{n+2}] + T_{n+1} C_1 \quad (4)$$

where

$$D_1 = \frac{C_2 + C_3}{1 + C_1} \quad \text{and} \quad D_2 = \frac{C_4}{1 + C_1}$$

$$D_1 = \frac{C_2 + C_3}{1 + C_1} \quad \text{and} \quad D_2 = \frac{C_4}{1 + C_1}$$

From equation (1.1)

$$T_{n+1} = D_1 T_n + D_2 [T_{n-1} + T_{n+2}] \quad (6)$$

where

$$D_1 = \frac{C_2 + C_3}{1 + C_1} \quad \text{and} \quad D_2 = \frac{C_4}{1 + C_1}$$

For the elements from the first and last row equation (1.2) becomes

$$T_1 - T_n = C_1 [T_1 + T_n - T_2 - T_{n-1}] \quad (1.5)$$

and

$$T_{n+1} - T_{n+2} = C_1 [T_{n+1} - T_{n+2} - T_n + T_{n-1}] \quad (1.6)$$

respectively. If the same procedure as before and solving for the matrix outlet temperatures, the following equations are obtained:

$$T_{1,1} = D_1 T_{1,0} + D_1 T_{1,1} + D_1 T_{1,2} + T_{1,3} \quad (3)$$

and

$$T_{N,1} = D_2 T_{N,0} + D_2 T_{N,1} + D_2 T_{N,2} + T_{N,3} \quad (5)$$

for the first and last row elements respectively, where

$$D_1 = \frac{C_p}{C_r} \quad \text{and} \quad D_2 = \frac{C_p}{C_r}$$

Equations (7) and (8) may be written as

$$T_{1,1} - T_{1,0} = \frac{C_p}{C_r} [T_{1,1} - T_{1,0} - T_{1,2} + T_{1,3}] \quad (1.7)$$

and

$$T_{N,1} - T_{N,0} = \frac{C_p}{C_r} [T_{N,1} - T_{N,0} - T_{N,2} + T_{N,3}] \quad (1.8)$$

Equation (1.3) reduces to

$$t_{n(N,g)} - t_{n(N+1,g)} = E_2 [t_{n(N,g+1)} - t_{n(N,g)}] + E_3 [t_{n(N,g)} + t_{n(N,g+1)} - t_{n(N-1,g)} - t_{n(N-1,g+1)}] \quad (1.9)$$

and

$$t_{1,g} - t_{1,g+1} = E_1 [t_{1,g+1} - t_{1,g}] + E_2 [t_{1,g} + t_{1,g+1} - t_{1,g-1} - t_{1,g-1+1}] \quad (1.10)$$

for the first and last row elements respectively, where

$$E = \frac{1}{C_{n,g}} \frac{1}{N} = NT \frac{1}{C_{n,g}} + \frac{1}{N} \frac{1}{C_{n,g}}$$

$$E_1 = \frac{1}{C_{1,g}} \frac{1}{N} \quad E_2 = \frac{1}{C_{n,g}} \frac{1}{N}$$

By comparison of equations (1.7) and (1.8) with equations (1.1) and (1.2) it follows that the form is the same and only the coefficients are different. The same holds true for equations (1.9) and (1.10) with equations (1.6) and (1.5) respectively. By substitution the following equations are obtained:

$$T_{n,g} = F_0 [t_{n(N,g)} - t_{n(N,g)}] + F_1 [t_{n(N,g)} + t_{n(N,g+1)} - t_{n(N-1,g)} - t_{n(N-1,g+1)}] \quad (9)$$

$$T_{1,g} = F_2 [t_{1,g+1} - t_{1,g}] + F_3 [t_{1,g} + t_{1,g+1} - t_{1,g-1} - t_{1,g-1+1}] \quad (10)$$

$$T_{n,g} = F_4 [t_{n,g} - t_{n,g+1}] + F_5 [t_{n,g} + t_{n,g+1} - t_{n-1,g} - t_{n-1,g+1}] \quad (11)$$

for the first, middle and last row elements respectively and

$$= \frac{1}{2} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right) \quad (12)$$

which is the same for all the elements, where

$$L_i = \frac{1}{2} \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} \right)$$

$$\frac{1}{\epsilon_1} = \frac{1}{\epsilon_2} = \frac{1}{\epsilon_1 + \epsilon_2}$$

$$\frac{1}{\epsilon_1} = \frac{1}{\epsilon_2} = \frac{1}{\epsilon_1 + \epsilon_2}$$

$$\frac{1}{\epsilon_1} = \frac{1}{\epsilon_2} = \frac{1}{\epsilon_1 + \epsilon_2}$$

$$\frac{1}{\epsilon_1} = \frac{1}{\epsilon_2} = \frac{1}{\epsilon_1 + \epsilon_2}$$

and

Convergence condition, -

Equation (1.2) can be put in the following form:

$$T_{max} = \frac{C_{min}}{C_{max}} T_{min} + \frac{C_{min}}{C_{max}} (T_{max} - T_{min}) \quad (1.11)$$

In the energy balance equations, the side of C_{max} was assumed to be the hot side so that by the second law of thermodynamics

and from equation (1.11)

$$T_{max} + T_{min} - (T_{max} + T_{min}) = T_{min} \quad (1.12)$$

Rearranging equation (1.12)

$$T_{max} \geq \frac{C_{min}}{C_{max}} T_{min} + \frac{C_{min}}{C_{max}} (T_{max} - T_{min}) \quad (1.13)$$

Again by the second law of thermodynamics the matrix temperature variation in the direction of fluid flow is such that

$$T_{x(0)} + T_{x(L)} = T_{max} = T_{min} \quad (1.14)$$

and

$$(1.15)$$

so that

$$(1.16)$$

Substituting equation (1.16) in equation (1.13)

$$(1.17)$$

Three cases are investigated:

1.- If $C_3 = C_2$ then equation (1.17) becomes

2.- If $C_3 = 0$ then equation (1.17) becomes

3.- If $C_2 - 2C_3 = 0$ then equation (1.17) becomes

From the above cases it can be seen that a sufficient condition for equation (2) to converge is that

Substituting the values for the constants and rearranging yields

For the first and last row elements the convergence criterion becomes

$$V_1 > \frac{1}{\epsilon}$$

which for substitution of the parameters gives

$$V_1 > \frac{1}{\epsilon}$$

This condition is less stringent than for the middle row elements.

Similarly, for the side o. Gain the sufficient condition for the middle row elements becomes

$$V_1 > \frac{1}{\epsilon}$$

which for substitution of the parameters gives

$$V_1 > \frac{1}{\epsilon}$$

For the first and last row elements it is found that

$$V_1 > \frac{1}{\epsilon}$$

which requires that

$$V_1 > \frac{1}{\epsilon}$$

is the sufficient condition. This again is a less stringent condition than that imposed by the middle row elements.

Differential equations.-

Hausen [4] has given a theory for regenerators without heat conduction. When the same assumptions are made, but the longitudinal conduction of heat is included, (See page 3) the differential equations obtained are [1] :

for the side of C_{\max} (gas flowing from $x = 0$ to $x = L$);

$$\left(\frac{dT}{dx} + \frac{T}{L} \right) = \frac{1}{2} (T - T_c) \quad (1)$$

$$- \frac{dT}{dx} = \frac{1}{2} (T - T_c) \quad (2)$$

and for the side of C_{\min} (reversed gas flow),

$$\left(\frac{dT}{dx} - \frac{T}{L} \right) = \frac{1}{2} (T - T_c) \quad (3)$$

$$\frac{dT}{dx} = \frac{1}{2} (T - T_c) \quad (4)$$

A temperature scale can be used for which the gas entrance temperature at the side of C_{\min} is zero and that at the side of C_{\max} is unity. The boundary conditions are then

and

$$\frac{dT}{dx} = \frac{dT}{dx} = 0 \quad (5)$$

APPENDIX 2

Computer program.-

The Fortran system was used to program the problem for the CDC 1604 digital computer.

The six dimensionless parameters are used as input together with increments and factors to change the values of NTU_0 , ϵ and As , so that solutions can be generated for several combinations of the parameters without having to feed into the computer these different sets of parameters every time.

Another input is the number of subdivisions of the three streams together with the increments, so that when the computer finishes the first series of runs for a given number of subdivisions it automatically increases the subdivisions by the specified increments and the two values of effectiveness obtained are extrapolated to an infinite number of subdivisions.

In order to obtain the initial estimates of matrix temperatures, the problem is first solved neglecting the longitudinal heat conduction as explained previously. For this case the equations derived in Appendix 1 are simplified to (for detailed derivation see Reference 1)

$$T_{1,n+1} = T_{1,n} - \Delta T_{1,n} - \Delta T_{1,n}^2$$

$$T_{2,n+1} = T_{2,n} - \Delta T_{2,n} - \Delta T_{2,n}^2$$

$$X_{11} = \frac{1}{2} \left(\frac{1}{\rho_0} - \frac{1}{\rho_0} \right)$$

$$Y_{11} = \frac{1}{2} \left(\frac{1}{\rho_0} - \frac{1}{\rho_0} \right)$$

where

$$A_1 = \left[1 + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} \right]^{-1}$$

$$A_2 = \left[1 + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} \right]^{-1}$$

$$A_3 = \left[1 + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} \right]^{-1}$$

$$A_4 = \left[1 + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} + \frac{1}{\frac{1}{\rho_0} - \frac{1}{\rho_0}} \right]^{-1}$$

The inlet matrix temperatures $T_{x(1,1)}$ necessary to initiate the problem for zero heat conduction are estimated by assuming $T_{x(1,1)} = T_{x(1,1)}$, (See Fig. 11) and feeding this value into the computer. The remaining inlet matrix temperatures are obtained from the relation

$$T_{x(i,j)} = \left[1 - \frac{C_{r(i,j)}}{C_{min}} \right] T_{x(i,j)}$$

The number of passes necessary to meet the desired accuracy in the solution changes accordingly with the initial estimates of matrix inlet temperatures, but considering that the computer time required to work the problem for zero heat conduction is so small as compared to the case with conduction, that these estimates do not become important. Only for high values of C_r/C_{min} these estimates may be of significance since for this case the change in the reversal condition after every pass is very small.

In general, it was found that an estimate $T_{x(1,1)}$ of
of 0.4 to 0.6 for $C_r/C_{min} < 5.0$
and 0.7 to 0.9 for $C_r/C_{min} > 5.0$

together with the above relation for $T_{x(i,j)}$ should converge to the solution in a reasonable time.

The temperature distribution is calculated by repetitive use of the iteration equations.

The effectiveness may be defined as

$$\epsilon = \frac{C_{max} (T_{xi} - T_{xo})}{C_{min} (T_{xi} - T_{ni})} = \frac{T_{no} - T_{ni}}{T_{xi} - T_{ni}} .$$

from the conditions of the problem

$$\bar{t}_{x_0} = \frac{1}{N_x} \sum_{j=1}^{N_x} \bar{t}_{x_0(j)} = 1$$

$$\bar{t}_{x_0} = \frac{1}{N_x} \sum_{j=1}^{N_x} t_{x_0(N+1-j)}$$

$$\bar{t}_{n_0} = \frac{1}{N_n} \sum_{j=1}^{N_n} t_{n_0(j)} = 0$$

$$\bar{t}_{n_0} = \frac{1}{N_n} \sum_{j=1}^{N_n} t_{n_0(N+1-j)}$$

therefore, the expression for effectiveness for the definition reduces to

$$\epsilon = \frac{1}{N_n} \sum_{j=1}^{N_n} t_{n_0(N+1-j)}$$

which is the expression used for the computation of the effectiveness.

The heat balance error is computed from the relation

$$\text{error} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{C_{n,m} (T_1 - T_2)}{C_{max} (T_1 - T_2)}$$

which reduces to

$$\text{error} = 1 - \frac{C_{n,m}}{C_{max}} = 1 - \frac{1}{\sum_{j=1}^{N_n} t_{n_0(N+1-j)}}$$

The extrapolated effectiveness is computed from the relation
(see Fig. 2)

$$E = (1 - E_1) + RE E_1$$

where

$$RE = \frac{N_1(N_2 + 1) - N_2(N_1 + 1)}{N_1(N_2 + 1) - N_2(N_1 + 1)}$$

The input parameters in Fortran notation are

$$P1 = C_{\min}/C_{\max}, P2 = C_r/C_{\min}, P3 = (hA)^*,$$

$$P4 = NTU_0, P5 = \lambda, P6 = As^*$$

The initial values of NTU_0 , λ and As^* are changed by the following relations

$$P4 = P4 + P4IN$$

$$P5 = P5 * P5FA + P5IN$$

$$P6 = P6 - P6IN$$

where $P4IN$, $P5FA$, $P5IN$, $P6IN$ are also input values. The inputs $P4FI$, $P5FI$ and $P6FI$ are the final values of $P4$, $P5$, and $P6$ respectively.

The program is written so that the computer after working the initial set of parameters, it reduces $P6$ by $P6IN$ every time until $P6$ becomes less than $P6FI$. Then it sets $P6$ equal to the initial value and changes $P5$. The procedure is again repeated until $P5$ becomes greater than $P5FI$. Then $P5$ is set equal to the initial value and $P4$ is increased by $P4IN$. The cycle is repeated until $P4$ becomes equal to or greater than $P4FI$.

During the computation, provisions are made by the use of Pause Statements and switches to make branch decisions in case the solution diverges or does not converge within the maximum number of passes specified. Figs. 7a and 7b show the complete flow diagram of the program which is followed by the Fortran listing of the program.

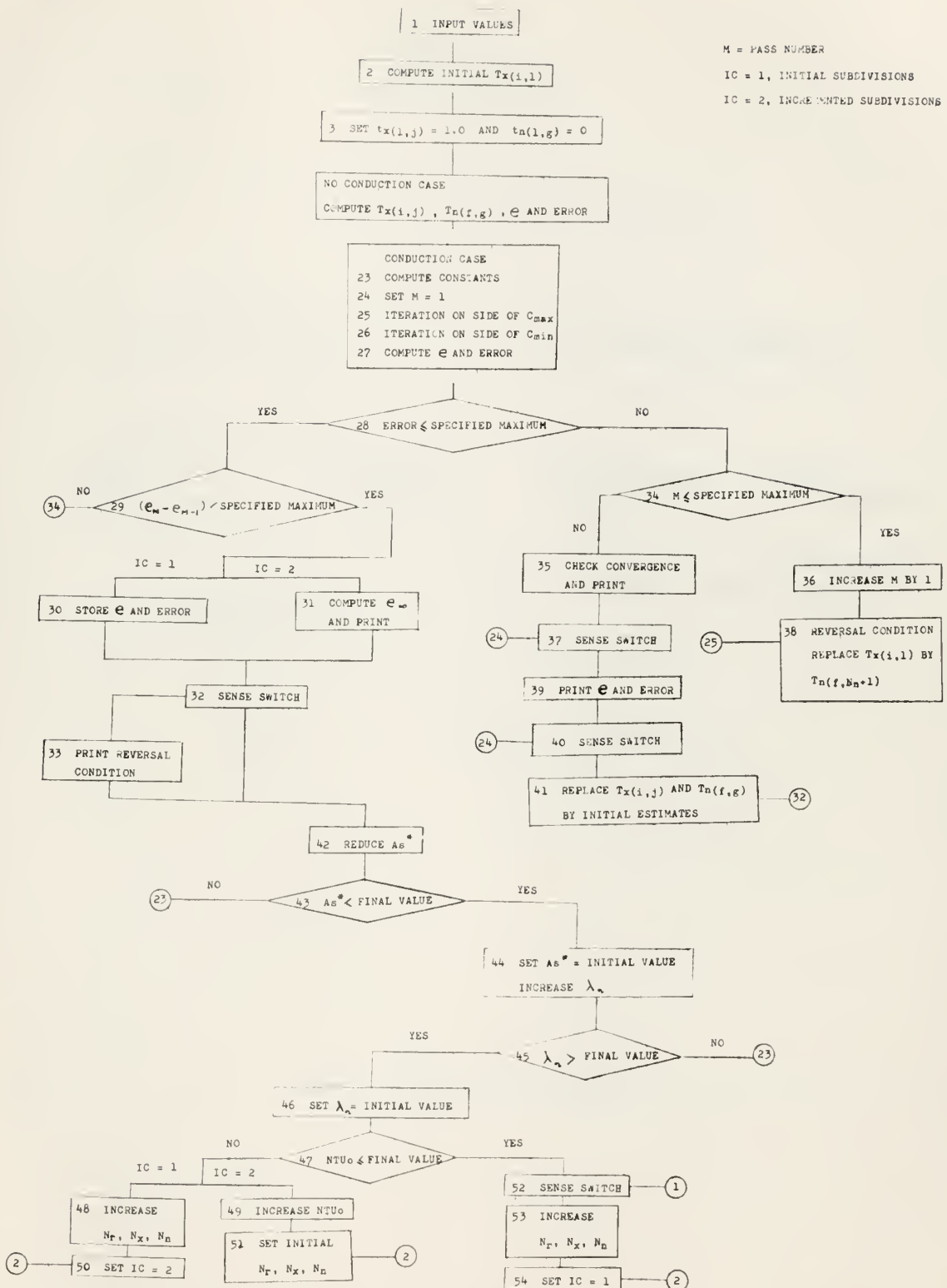


FIG. 7a GENERAL FLOW DIAGRAM

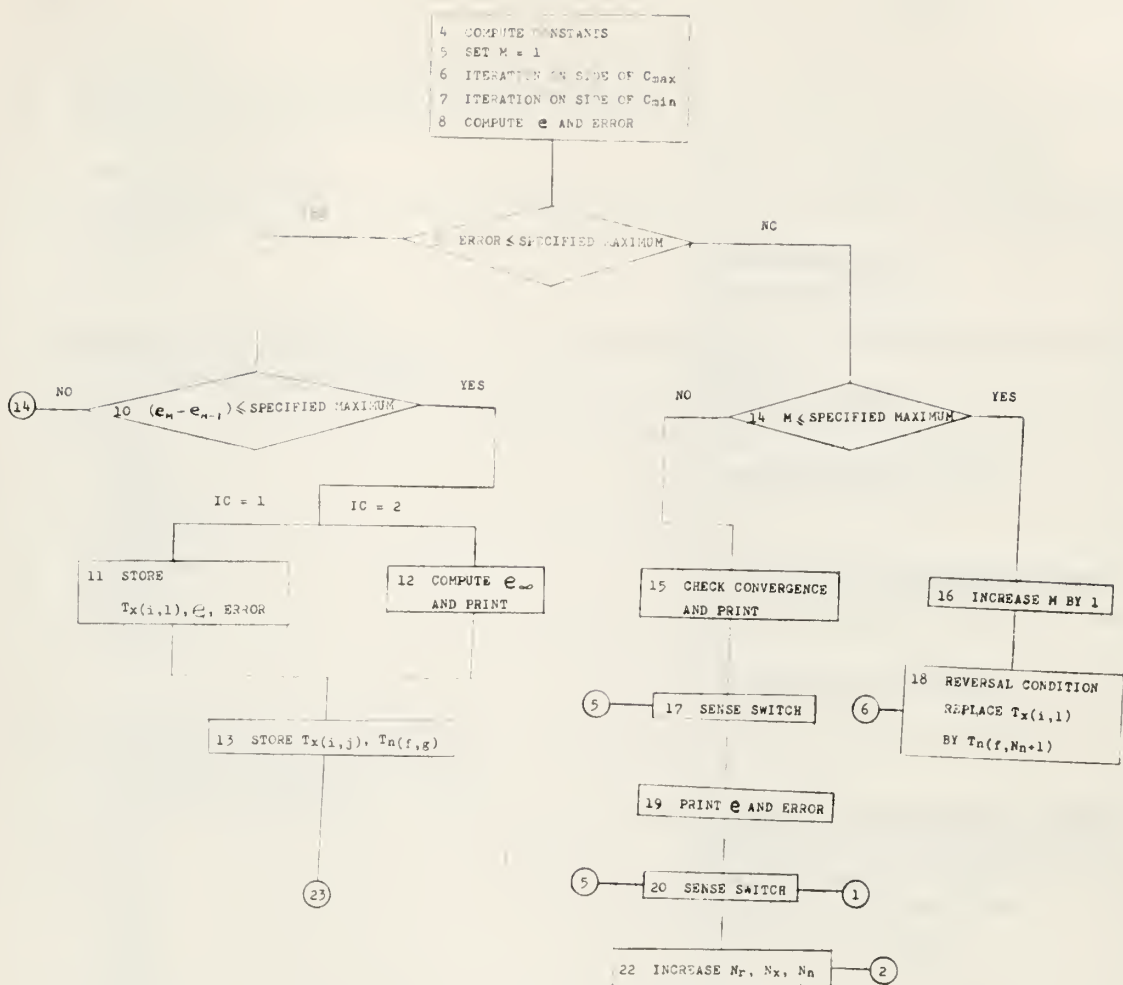


FIG. 7b FLOW DIAGRAM FOR NO CONDUCTION

PROGRAM	REMARKS
PROGRAM BAHNKE DIMENSION TH(40,40),UH(40,40),TC(40,40),UC(40,40),ERR(92),TCO(92), IUGC(40,40),UCC(40,40),UH1(40),UH2(40),EF(20,20),FR(20,20)	
1 FORMAT(6F5.2) READ 1,P1,P2,P3,P4,P4IN,P4F1 2 FORMAT(6I5) READ 2,NM,NH,NC,NM1,NH1,NC1	INPUT $NM = N_T, NH = N_X, NC = N_D$
3 FORMAT(F5.3) READ 3,UH(1,1) EM=NM DO 401 K=1,NM E=K 401 UH(K,1)=[(EM+1.0-E)*UH(1,1)]/EM	INPUT: $UH(1,1) = T_x(1,1)$ ESTIMATES OF $T_x(1,1)$
4 FORMAT(7F7.2) READ 4,P5,P5IN,P5FA,P5F1,P6,P6IN,P6F1 S5=P5 S6=P6	INPUT STORE INITIAL P5 AND P6
5 NM2=NM+NM1 NH2=NH+NH1 NC2=NC+NC1	NUMBER OF SUBDIVISIONS FOR 2nd RUN
NE1=NM*(NH+NC1) RE1=NE1 NE2=NM2*(NH2+NC2) RE2=NE2 RE=RE1/(RE2-RE1)	RE = VARIABLE USED TO EXTRAPOLATE EFFECTIVENESS
EM2=NM2 DO6 L=1,NM2 E=L RT=1.0-((E-1.0)/EM2)	ESTIMATES OF $T_x(1,1)$ FOR 2nd RUN
6 UH2(L)=UH(1,1)*RT 7 FORMAT(//14HNO OF ELEMENTS3X,3I5,4X,3I5) WRITE OUTPUT TAPE 2,7,NM,NH,NC,NM2,NH2,NC2 PRINT 7,NM,NH,NC,NM2,NH2,NC2	
8 ICG=1	INDICATES 1st RUN, USING INITIAL No OF SUBDIV.
9 EM=NM EH=NH EC=NC	FIXED TO FLOATING POINT CONVERSION
NMM=NM-1 NHP=NH-1 NCP=NC-1 NMP=NM+1	DEFINITION OF NEW VARIABLES
10 DO 10 J=1,NH TH(1,J)=1.0 DO 11 L=1,NC 11 TC(1,L)=0.0	SET $t_{x(1,j)} = 1.0$ AND $t_{n(1,g)} = 0.0$
12 A10=(1.0+EM/(P1*P2*EH)+(2.0*EM*P3)/(P4*P1*(1.0+P3)) A1=2.0/A10 A20=1.0*(P1*P2*EH)/EM+(2.0*EH*P3*P2)/(P4*(1.0+P3)) A2=2.0/A20 A30=1.0*EM/(P2*EC)+(2.0*EM)/(P4*(1.0+P3)) A3=2.0/A30 A40=1.0*(P2*EC)/EM+(2.0*EC*P2)/(P4*(1.0+P3)) A4=2.0/A40	CONSTANTS FOR THE NO CONDUCTION CASE
C HOT SIDE NO CONDUCTION 121 M=1	M DENOTES PASS NUMBER
13 DO 15 J=1,NH DO 15 I=1,NM TH(I+1,J)=TH(I,J)+A1*(UH(I,J)-TH(I,J)) 15 UH(I,J+1)=UH(I,J)-A2*(UH(I,J)-TH(I,J))	ITERATION ON SIDE OF C_{max}
C COOL SIDE NO CONDUCTION DO 17 K=1, NM 17 UC(K,1)=UH(NM+1-K,NH+1) DO 19 L=1,NC DO19 K=1,NM TC(K+1,L)=TC(K,L)+A3*(UC(K,L)-TC(K,L)) 19 UC(K,L+1)=UC(K,L)-A4*(UC(K,L)-TC(K,L))	ITERATION ON SIDE OF C_{min}
G1=0.0 DO 20 J=1,NH 20 G1=TH(NH+1,J)+G1 THO=G1/LH	$THO = [t_{x(N_T+1,j)}]_{avg}$

	G2=0.0 DO 21 L=1,NM G2=TC(NM+1,L)+G2 TCO(M)=G2/EC	TCC = EFFECTIVENESS
21	ERR(M)=1.0-(P1*TCO(M+1)/(1.0-THO)) IF(ERR(M)) 23,24,24	
23	ERR(M)=-ERR(M)	COMPUTE HEAT BALANCE ERROR AND COMPARE IT WITH MAXIMUM SPECIFIED
24	IE(ERR(M)-4.0E-6) 30,30,25	
25	IF(90-M) 26,26,215	
26	IF(ERR(M-1)-ERR(M)) 29,29,27	
27	PAUSE 11	CHECK CONVERGENCE PAUSE 11 = SOLUTION CONVERGES
28	IF(SENSE SWITCH 1) 32,121	PAUSE 12 = SOLUTION DIVERGES
29	PAUSE 12 GO TO 28	
215	DO 22 I=1,NM	
22	UH(I,1)=UC(NM+1-I,NC+1) M=M+1 GO TO 13	REVERSAL CONDITION
30	T2=TCO(M-1)-TCO(M) IF(T2) 301,302,302	
301	T2=-T2	CHECK DIFFERENCE IN EFFECTIVENESS IN THE LAST TWO PASSES
302	IF(T2-4.0E-6) 305,305,25	
305	IF(IC-1) 31,31,33	
31	DO 312 J=1,NM	
312	UH1(J)=UH(J,1) EF(1,1)=TCO(M) ER(1,1)=ERR(M) GO TO 39	STORE $T_x(i,1)$, EFFECTIVENESS AND ERROR FOR THE 1st RUN
32	FORMAT(4F8.2,(5,F10.6,F9.2,13) PRINT 32,P1,P2,P3,P4,M,TCO(M),ERR(M),IC PAUSE 13 IF(SENSE SWITCH 1) 321,121	
321	IF(SENSE SWITCH 2) 1,322	
322	NM=NM2 NH=NH2 NC=NC2 GO TO 5	INCREASE NO. OF SUBDIVISIONS AND REPEAT 1st RUN
33	EFF=TCO(M)*(1.0+RE)-EF(1,1)+RE B1=EFF	EXTRAPOLATE EFFECTIVENESS AND STORE
34	FORMAT(//4F8.2,16X,2(F10.6,F9.2),F10.6) WRITE OUTPUT TAPE 2,34,P1,P2,P3,P4,EF(1,1),ER(1,1),TCO(M),ERR(M), IEFF	
35	DO 35 L=1,NM UH2(L)=UH(L,1)	STORE $T_x(i,1)$ FROM 2nd RUN
39	IA=2 IB=1	SUBSCRIPTS FOR ERROR AND EFFECTIVENESS
40	DO 44 I=1,NM	
41	DO 42 J=1,NHP	
42	UHC(I,J)=UH(I,J)	
43	DO 44 L=1,NCP	
44	UCC(I,L)=UC(I,L)	$UHC(I,J) = T_x(i,j)$, $UCC(I,L) = T_a(f,g)$ FOR CONDUCTION CASE
C	CONSTANTS	
50	E1=(P4*(1.0+P3))/(2.0*EM) E11=1.0+E1 F2=(P2*EC)/EM E3=0.5*P5*EM E4=E1+(E11*(2.0+E3+E21)) E5=E1+(E11*(E2+E3)) F1=(2.0*E11)/E4 F2=(E1+(E11*(2.0+E3-E21)))/E4 F3=(E3*E11)/E4 F4=(1.0-E11)/E11 F5=E1/E11 F6=(2.0*E11)/E5 F7=(E1+(E11*(E3-E21)))/E5 F8=(E3*E11)/E5 C1=(P1*P4*(1.0+P3))/(2.0*P3*EM) C11=1.0+C1 C2=(P1*P2*EM)/EM C3=(P1*P5*EM)/(2.0*P6) C4=C1+(C11*(C2+2.0*C3)) C5=C1+(C11*(C2+C3))	CONSTANTS FOR CONDUCTION CASE

	D1=(2.0*C1)/C4 D2=(C1+(C11*(2.0*C3-C2)))/C4 D3=(C3*C11)/C4 D4=(1.0-C1)/C11 D5=C1/C11 D6=(2.0*C11/C5 D7=(C1+(C11*(C3-C2)))/C5 D8=(C3*C11)/C5	CONSTANTS FOR CONDUCTION CASE
53 C	M=1 HOT SIDE WITH CONDUCTION DO 67 J=1,NH L=)	
60	61 UHC(I,J)=D6*TH(I,J)-D7*UHC(I,J)+D8*(UHC(2,J)+UHC(2,J+1)) TH(2,J)=D4*TH(1,J)+D5*(UHC(1,J)+UHC(1,J+1)) 62 DO 64 I=2,NMH UHC(I,J+1)=D1*TH(I,J)-D2*UHC(I,J)+D5*(UHC(I-1,J)+UHC(I-1,J+1)+ UHC(I+1,J)+UHC(I+1,J+1)) 64 TH(I+1,J)=D4*TH(I,J)+D5*(UHC(I,J)+UHC(I,J+1)) UHC(NM,J+1)=D6*TH(NM,J)-D7*UHC(NM,J)+D8*(UHC(NMM,J)+UHC(NMM,J+1)) 66 TH(NMP,J)=D4*TH(NM,J)+D5*(UHC(NM,J)+UHC(NM,J+1)) L=L+1)F(3-L) 67,67,61	ITERATION ON SIDE OF C _{max} TWO PASSES PER COLUMN
67	CONTINUE	
68	DO 69 K=1,NM	
69	UCC(K,1)=UHC(NMP-K,NMP)	
C	COLO SIDE WITH CONDUCTION	
70	DO 75 L=1,NC J=1	
71	UCC(1,L+1)=F6*TC(1,L)-F7*UCC(1,L)+F8*(UCC(2,L)+UCC(2,L+1)) TC(2,L)=F4*TC(1,L)+F5*(UCC(1,L)+UCC(1,L+1))	
72	DO 73 K=2,NMM UCC(K,L+1)=F1*TC(K,L)-F2*UCC(K,L)+F5*(UCC(K-1,L)+UCC(K-1,L+1)+ UCC(K+1,L)+UCC(K+1,L+1)) 73 TC(K+1,L)=F4*TC(K,L)+F5*(UCC(K,L)+UCC(K,L+1)) UCC(NM,L+1)=F6*TC(NM,L)-F7*UCC(NM,L)+F8*(UCC(NMM,L)+UCC(NMM,L+1)) 74 TC(NMP,L)=F4*TC(NM,L)+F5*(UCC(NM,L)+UCC(NM,L+1)) J=J+1 IF(3-J) 75,75,71	ITERATION ON SIDE OF C _{min} TWO PASSES PER COLUMN
75	CONTINUE	
	G1=0.0 DO 76 J=1,NH	
76	G1=TH(NMP,J)+G1 TH0=G1/H	
	G2=0.0 DO 78 L=1,NC	
78	G2=TC(NMP,L)+G2 TC0(M)=G2/FC	
	ERR(M)=1.0-(P)*TC0(M)/(1.0-TH0) IF(ERR(M)) 80,80,81	
80	ERR(M)=-ERR(M)	
81	IF(ERR(M)-4.0E-6) 90,90,82	
82	IF(90-M) 83,83,85	
83	IF(ERR(M)-1)-ERR(M)) 84,84,87	
84	PAUSE 22	
841	IF(SENSE SWITCH 1) 842,53	
842	IF(SENSE SWITCH 2) 88,884	
85	M=M+1 DO 86 I=1,NM	
86	UHC(I,1)=UCC(NMP-I,NCP) GO TO 60	
87	PAUSE 21 GO TO 841	
88	IF(IC-) 881,881,882	
881	EF(IA,18)=0.0 ER(IA,18)=0.0 GO TO 97	
882	FORMAT(11HSQL DIV FOR13,18X,E9.2,F8.2,2(F10.6,F9.2)) WRITE OUTPUT TAPE 2,882,IC,P5,P6,FF(IA,18),ER(IA,18),TC0(M),ERR(M) GO TO 97	

```

884  FORMAT(4F8.2,E8.2,F8.2,2X,13,F10.6,E9.2)
      PRINT 884,P1,P2,P3,P4,P5,P6,IC,TCO(M),ERR(M)
      PAUSE 23
      IF(SENSE SWITCH 1) 885,53
885  IF(SENSE SWITCH 2) 114,82
89  IF(IC-1) 891,891,93
891  EF(IA,IB)=TCO(M)
      ER(IA,IB)=ERR(M)
      GO TO 102
90  T1=TCO(M-1)-TCO(M)
      IF(T11 91,92,92
91  T1=-T1
92  IF(T1-4.0E-61 89,89,82
93  EFF=TCO(M)*(1.0+RE)-FF(IA,IP)*RE
      B2=((IB)-EFF)*100.0/R1
94  FORMAT(32X,E8.2,F8.2,2(F10.6,E9.2),F10.6,F8.3)
      WRITE OUTPUT TAPE 2,94,P5,P6,EF(IA,IR),ER(IA,IB),TCO(M),ERR(M),EFF
      1,82
      IF(SENSE SWITCH 3) 95,102
95  FORMAT(F8.2,E9.2,F8.4)
      PRINT 95,P4,P5,B2
      GO TO 102
97  DO 974 I=1,NM
971  DO 972 J=1,NHP
972  UHC(I,J)=UH(I,J)
973  DO 974 L=1,NCP
974  UCC(I,L)=UC(I,L)
102  IF(SENSE SWITCH 3) 131,132
131  FORMAT(1/4F8.2,E8.2,F8.2,3X,13/(15F7.4))
      WRITE OUTPUT TAPE 3,131,P1,P2,P3,P4,P5,P6,IC,
      1(UHC(J,1),UCC(NMP-J,NCP),J=1,NM)
132  P6=P6-P6IN
      IB=IB+1
      IF(P6-P6F1) 103,103,51
103  P6=S6
      IB=1
104  P5=P5+P5FA+P5IN
      IA=IA+1
      IF(P5F1-P51 105,50,50
105  P5=S5
      IF(IC-1) 106,106,108
106  IC=2
      NM=NM2
      NH=NH2
      NC=NC2
      DO 107 K=1,NM
107  UH(K,1)=UH2(K)
      GO TO 9
108  IF(P4F1-P41 113,113,109
109  P4=P4+P4IN
      NM=NM-NM1
      NH=NH-NH1
      NC=NC-NC1
      DO 111 K=1,NM
111  UH(K,1)=UH1(K)
      GO TO 8
113  PAUSE 50
      IF(SENSE SWITCH 1) 114,1
114  DO 115 L=1,NM
115  UH(L,1)=UH2(L)
      GO TO 5
      ENO
      ENO

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